

## UK SENIOR MATHEMATICAL CHALLENGE

Tuesday 8 November 2016

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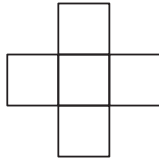
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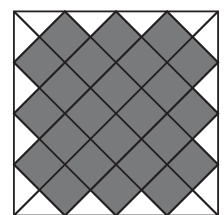
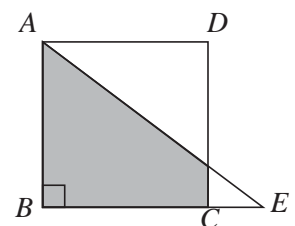
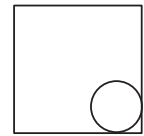
### **RULES AND GUIDELINES** (to be read before starting)

1. Do not open the question paper until the invigilator tells you to do so.
2. Time allowed: **90 minutes**.  
No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
3. The use of rough paper is allowed.  
**Calculators, measuring instruments and squared paper are forbidden.**
4. Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
5. **Use B or HB pencil only.** Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
6. **Scoring rules:** all candidates start out with 25 marks;  
0 marks are awarded for each question left unanswered;  
4 marks are awarded for each correct answer;  
**1 mark is deducted** for each incorrect answer.
7. **Guessing:** Remember that there is a penalty for incorrect answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

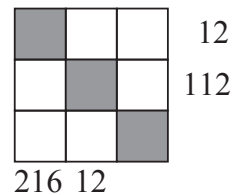
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*<http://www.ukmt.org.uk>*

1. How many times does the digit 9 appear in the answer to  $987654321 \times 9$  ?  
 A 0                      B 1                      C 5                      D 8                      E 9
2. On a Monday, all prices in Isla's shop are 10% more than normal. On Friday all prices in Isla's shop are 10% less than normal. James bought a book on Monday for £5.50. What would be the price of another copy of this book on Friday?  
 A £5.50                  B £5.00                  C £4.95                  D £4.50                  E £4.40
3. The diagram shows a circle with radius 1 that rolls without slipping around the inside of a square with sides of length 5. The circle rolls once around the square, returning to its starting point. What distance does the centre of the circle travel?  
 A  $16 - 2\pi$               B 12                      C  $6 + \pi$                   D  $20 - 2\pi$               E 20
4. Alex draws a scalene triangle. One of the angles is  $80^\circ$ . Which of the following could be the difference between the other two angles in Alex's triangle?  
 A  $0^\circ$                       B  $60^\circ$                       C  $80^\circ$                       D  $100^\circ$                       E  $120^\circ$
5.  All the digits 2, 3, 4, 5 and 6 are placed in the grid, one in each cell, to form two three-digit numbers that are squares. Which digit is placed in the centre of the grid?  
 A 2                      B 3                      C 4                      D 5                      E 6
6. The diagram shows a square  $ABCD$  and a right-angled triangle  $ABE$ . The length of  $BC$  is 3. The length of  $BE$  is 4. What is the area of the shaded region?  
 A  $5\frac{1}{4}$                   B  $5\frac{3}{8}$                       C  $5\frac{1}{2}$                       D  $5\frac{5}{8}$                       E  $5\frac{3}{4}$
7. Which of these has the smallest value?  
 A  $2016^{-1}$                   B  $2016^{-1/2}$                   C  $2016^0$                       D  $2016^{1/2}$                       E  $2016^1$
8. Points are drawn on the sides of a square, dividing each side into  $n$  equal parts (so, in the example shown,  $n = 4$ ). The points are joined in the manner indicated, to form several small squares (24 in the example, shown shaded) and some triangles.  
 How many small squares are formed when  $n = 7$  ?  
 A 56                      B 84                      C 140                      D 840                      E 5040
9. A square has vertices at  $(0, 0)$ ,  $(1, 0)$ ,  $(1, 1)$  and  $(0, 1)$ . Graphs of the following equations are drawn on the same set of axes as the square.  
 $x^2 + y^2 = 1$ ,       $y = x + 1$ ,       $y = -x^2 + 1$ ,       $y = x$ ,       $y = \frac{1}{x}$   
 How many of the graphs pass through exactly two of the vertices of the square?  
 A 1                      B 2                      C 3                      D 4                      E 5



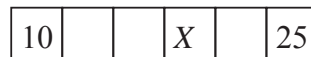
10. The digits from 1 to 9 are to be written in the nine cells of the  $3 \times 3$  grid shown, one digit in each cell.



The product of the three digits in the first row is 12.  
 The product of the three digits in the second row is 112.  
 The product of the three digits in the first column is 216.  
 The product of the three digits in the second column is 12.  
 What is the product of the digits in the shaded cells?

- A 24                  B 30                  C 36                  D 48                  E 140

11. In the grid below each of the blank squares and the square marked  $X$  are to be filled by the mean of the two numbers in its adjacent squares. Which number should go in the square marked  $X$ ?

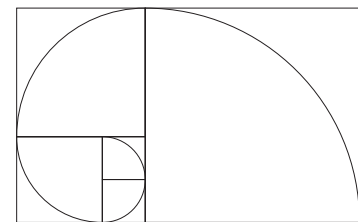


- A 15                  B 16                  C 17                  D 18                  E 19

12. What is the smallest square that has 2016 as a factor?

- A  $42^2$                   B  $84^2$                   C  $168^2$                   D  $336^2$                   E  $2016^2$

13. Five square tiles are put together side by side. A quarter circle is drawn on each tile to make a continuous curve as shown. Each of the smallest squares has side-length 1. What is the total length of the curve?



- A  $6\pi$                   B  $6.5\pi$                   C  $7\pi$                   D  $7.5\pi$                   E  $8\pi$

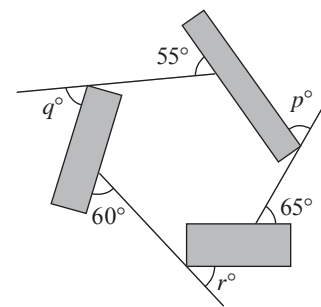
14. Which of the following values of the positive integer  $n$  is a counterexample to the statement: "If  $n$  is not prime then  $n - 2$  is not prime"?

- A 6                  B 11                  C 27                  D 33                  E 51

15. The diagram shows three rectangles and three straight lines.

What is the value of  $p + q + r$ ?

- A 135                  B 180                  C 210  
 D 225                  E 270



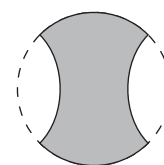
16. For which value of  $k$  is  $\sqrt{2016} + \sqrt{56}$  equal to  $14^k$ ?

- A  $\frac{1}{2}$                   B  $\frac{3}{4}$                   C  $\frac{5}{4}$                   D  $\frac{3}{2}$                   E  $\frac{5}{2}$

17. Aaron has to choose a three-digit code for his bike lock. The digits can be chosen from 1 to 9. To help him remember them, Aaron chooses three different digits in increasing order, for example 278. How many such codes can be chosen?

- A 779                  B 504                  C 168                  D 84                  E 9

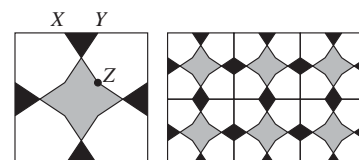
18. The circumference of a circle with radius 1 is divided into four equal arcs. Two of the arcs are 'turned over' as shown. What is the area of the shaded region?



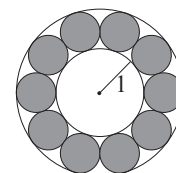
- A 1      B  $\sqrt{2}$       C  $\frac{1}{2}\pi$       D  $\sqrt{3}$       E 2
19. Let  $S$  be a set of five different positive integers, the largest of which is  $m$ . It is impossible to construct a quadrilateral with non-zero area, whose side-lengths are all distinct elements of  $S$ . What is the smallest possible value of  $m$ ?

- A 2      B 4      C 9      D 11      E 12
20. Michael was walking in Marrakesh when he saw a tiling formed by tessellating the square tile as shown.

The tile has four lines of symmetry and the length of each side is 8 cm. The length of  $XY$  is 2 cm. The point  $Z$  is such that  $XZ$  is a straight line and  $YZ$  is parallel to sides of the square. What is the area of the central grey octagon?



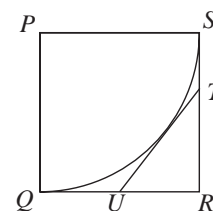
- A  $6 \text{ cm}^2$       B  $7 \text{ cm}^2$       C  $8 \text{ cm}^2$       D  $9 \text{ cm}^2$       E  $10 \text{ cm}^2$
21. The diagram shows ten equal discs that lie between two concentric circles – an inner circle and an outer circle. Each disc touches two neighbouring discs and both circles. The inner circle has radius 1. What is the radius of the *outer* circle?



- A  $2 \tan 36^\circ$       B  $\frac{\sin 36^\circ}{1 - \sin 36^\circ}$       C  $\frac{1 + \sin 18^\circ}{1 - \sin 18^\circ}$       D  $\frac{2}{\cos 18^\circ}$       E  $\frac{9}{5}$
22. Three friends make the following statements.  
Ben says, "Exactly one of Dan and Cam is telling the truth."  
Dan says, "Exactly one of Ben and Cam is telling the truth."  
Cam says, "Neither Ben nor Dan is telling the truth."  
Which of the three friends is lying?

- A Just Ben      B Just Dan      C Just Cam      D Each of Ben and Cam  
E Each of Ben, Cam and Dan
23. A cuboid has sides of lengths 22, 2 and 10. It is contained within a sphere of the smallest possible radius. What is the side-length of the largest cube that will fit inside the same sphere?

- A 10      B 11      C 12      D 13      E 14
24. The diagram shows a square  $PQRS$ . The arc  $QS$  is a quarter circle. The point  $U$  is the midpoint of  $QR$  and the point  $T$  lies on  $SR$ . The line  $TU$  is a tangent to the arc  $QS$ . What is the ratio of the length of  $TR$  to the length of  $UR$ ?



- A 3 : 2      B 4 : 3      C 5 : 4      D 7 : 6      E 9 : 8
25. Let  $n$  be the smallest integer for which  $7n$  has 2016 digits. What is the units digit of  $n$ ?
- A 0      B 1      C 4      D 6      E 8