United Kingdom
Mathematics Trust

Intermediate Mathematical Olympiad

Cayley paper

Thursday 16 March 2023

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England & Wales: Year 9 or below
Scotland: S2 or below
Northern Ireland: Year 10 or below

These problems are meant to be challenging! The earlier questions tend to be easier; later questions tend to be more demanding.

Do not hurry, but spend time working carefully on one question before attempting another.

Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. Start each question on an official answer sheet on which there is a QR code.
5. If you use additional sheets of (plain or lined) paper for a question, please write the following in the top left-hand corner of each sheet. (i) The question number. (ii) The page number for that question. (iii) The digits following the ':' from the question's answer sheet QR code. Please do not write your name or initials on additional sheets. **Do not hand in rough work**.
6. Your answers should be fully simplified, and exact. They may contain symbols such as π, fractions, or square roots, if appropriate, but not decimal approximations.
7. You should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

challenges@ukmt.org.uk

www.ukmt.org.uk

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◇ *Try to finish whole questions even if you cannot do many.*

◇ *You will have done well if you hand in full solutions to two or more questions.*

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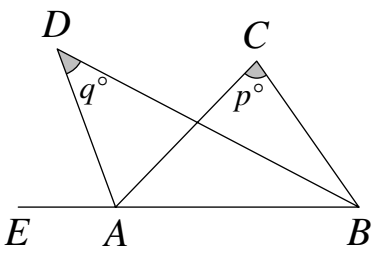
◇ *Incomplete or poorly presented solutions will not receive full marks.*

◇ *Do* not *hand in rough work.*

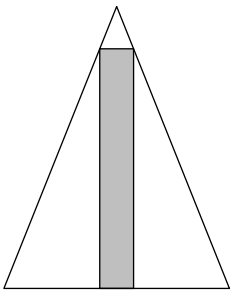
1. A four-digit number, n, is written as '$ABCD$' where A, B, C and D are all different odd digits. It is divisible by each of A, B, C and D. Find all the possible numbers for n.

2. The diagram shows a triangle ABC with side BA extended to a point E. The bisector of $\angle ABC$ meets the bisector of angle $\angle EAC$ at D. Let $\angle BCA = p^\circ$ and $\angle BDA = q^\circ$. Prove that $p = 2q$.



3. Aroon's PIN has four digits. When the first digit (reading from the left) is moved to the end of the PIN, the resulting integer is 6 less than 3 times Aroon's PIN. What could Aroon's PIN be?

4. The diagram shows a rectangle inside an isosceles triangle. The base of the triangle is n times the base of the rectangle, for some integer n greater than 1. Prove that the rectangle occupies a fraction $\frac{2}{n} - \frac{2}{n^2}$ of the total area.



5. The whole numbers from 1 to $2k$ are split into two equal-sized groups in such a way that any two numbers from the same group share no more than two distinct prime factors. What is the largest possible value of k?

6. A bag contains 7 red discs, 8 blue discs and 9 yellow discs. Two discs are drawn at random from the bag. If the discs are the same colour then they are put back into the bag. However, if the discs are different colours then they are removed from the bag and a disc of the third colour is placed in the bag. This procedure is repeated until there is only one disc left in the bag or the only remaining discs in the bag have the same colour. What colour is the last disc (or discs) left in the bag?

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