United Kingdom
Mathematics Trust

Senior Mathematical Challenge

Tuesday 6th November 2018

Organised by the United Kingdom Mathematics Trust

Supported by Institute and Faculty of Actuaries

Candidates must be full-time students at secondary school or FE college.
England & Wales: *Year* 13 *or below*
Scotland: *S*6 *or below*
Northern Ireland: *Year* 14 *or below*

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **90 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:** all candidates start with 25 marks;
   0 marks are awarded for each question left unanswered;
   4 marks are awarded for each correct answer;
   1 mark is deducted for each incorrect answer (to discourage guessing).
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Senior Mathematical Challenge should be sent to:

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1. When the following are evaluated, how many of the answers are odd numbers?

$$1^2,\ 2^3,\ 3^4,\ 4^5,\ 5^6$$

A 1 B 2 C 3 D 4 E 5

2. The positive integer 2018 is the product of two primes.

What is the sum of these two primes?

A 1001 B 1010 C 1011 D 1100 E 1101

3. Which of the following shows the digit 6 after it has been rotated clockwise through 135°?

A B C D E

4. Which of the following is not a multiple of 5?

A $2019^2 - 2014^2$ B $2019^2 \times 10^2$ C $2020^2 \div 101^2$ D $2010^2 - 2005^2$
E $2015^2 \div 5^2$

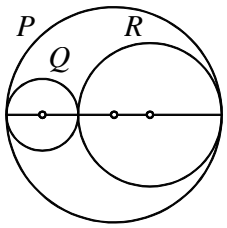
5. Which of the following numbers is the largest?

A $\frac{397}{101}$ B $\frac{487}{121}$ C $\frac{596}{153}$ D $\frac{678}{173}$ E $\frac{796}{203}$

6. Which of the following is equal to $25 \times 15 \times 9 \times 5.4 \times 3.24$?

A 3^9 B 3^{10} C 3^{11} D 3^{14} E 3^{17}

7. The circles P, Q and R are all tangent to each other. Their centres all lie on a diameter of P, as shown in the figure.



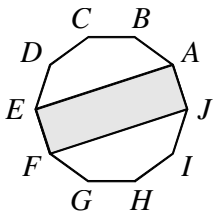
What is the value of $\dfrac{\text{circumference of } Q + \text{circumference of } R}{\text{circumference of } P}$?

A 1 B $\frac{1}{2}$ C $\frac{1}{3}$ D $\frac{1}{4}$
E more information needed

8. What are the last two digits of 7^{2018}?

A 07 B 49 C 43 D 01 E 18

9. The diagram shows a rectangle $AEFJ$ inside a regular decagon $ABCDEFGHIJ$.



What is the ratio of the area of the rectangle to the area of the decagon?

A 2 : 5 B 1 : 4 C 3 : 5 D 3 : 10 E 3 : 20

10. On a training ride, Laura averages speeds of 12 km/h for 5 minutes, then 15 km/h for 10 minutes and finally 18 km/h for 15 minutes.

What was her average speed over the whole ride?

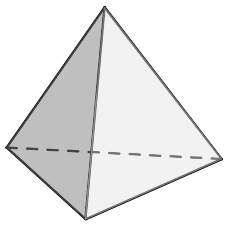
A 13 km/h B 14 km/h C 15 km/h D 16 km/h E 17 km/h

11. How many of the following four equations has a graph that does *not* pass through the origin?

$$y = x^4 + 1 \qquad y = x^4 + x \qquad y = x^4 + x^2 \qquad y = x^4 + x^3$$

A 0 B 1 C 2 D 3 E 4

12. A regular tetrahedron is a polyhedron with four faces, each of which is an equilateral triangle, as shown. A solid regular tetrahedron is cut into two pieces by a single plane cut.



Which of the following could *not* be the shape of the section formed by the cut?

A a pentagon
B a square
C a rectangle that is not a square
D a trapezium
E a triangle that is not equilateral

13. The lines $y = x$ and $y = mx - 4$ intersect at the point P.

What is the sum of the positive integer values of m for which the coordinates of P are also positive integers?

A 3 B 5 C 7 D 8 E 10

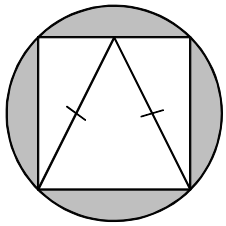
14. The following twelve integers are written in ascending order:

$$1, x, x, x, y, y, y, y, y, 8, 9, 11.$$

The mean of these twelve integers is 7. What is the median?

A 6 B 7 C 7.5 D 8 E 9

15. A square is inscribed in a circle of radius 1. An isosceles triangle is inscribed in the square as shown.



What is the ratio of the area of this triangle to the area of the shaded region?

A $\pi : \sqrt{2}$ B $\pi : 1$ C $1 : 4$ D $1 : \pi - 2$ E $2 : \pi$

16. The numbers p, q, r and s satisfy the following equations:

$$p + 2q + 3r + 4s = k \qquad 4p = 3q = 2r = s.$$

What is the smallest value of k for which p, q, r and s are all positive integers?

A 20 B 24 C 25 D 77 E 154

17. Bethany has 11 pound coins and some 20p coins and some 50p coins in her purse. The mean value of the coins is 52 pence.

Which could not be the number of coins in the purse?

A 35 B 40 C 50 D 65 E 95

18. P, Q and R are the three angles of a triangle, when each has been rounded to the nearest degree.

Which of the following is the complete list of possible values of $P + Q + R$?

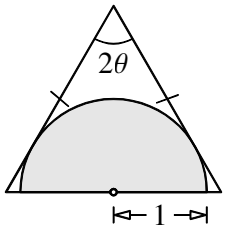
A 179°, 180° or 181° B 180°, 181° or 182° C 178°, 179° or 180° D 180°
E 178°, 179°, 180°, 181° or 182°

19. How many pairs of numbers (m, n) are there such that the following statement is true?

'A regular m-sided polygon has an exterior angle of size $n°$ and a regular n-sided polygon has an exterior angle of size $m°$.'

A 24 B 22 C 20 D 18 E 16

20. The diagram shows a semicircle of radius 1 inside an isosceles triangle. The diameter of the semicircle lies along the 'base' of the triangle, and the angle of the triangle opposite the 'base' is equal to 2θ. Each of the two equal sides of the triangle is tangent to the semicircle.



What is the area of the triangle?

A $\frac{1}{2}\tan 2\theta$ B $\sin\theta\cos\theta$ C $\sin\theta + \cos\theta$ D $\frac{1}{2}\cos 2\theta$

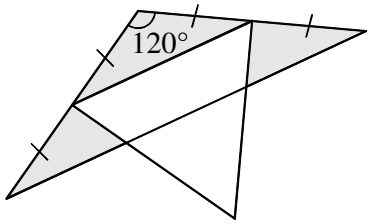
E $\dfrac{1}{\sin\theta\cos\theta}$

21. The graph of $y = \dfrac{1}{x}$ is reflected in the line $y = 1$. The resulting image is reflected in the line $y = -x$.

What is the equation of the final graph?

A $y = \dfrac{-1}{(x+2)}$ B $y = \dfrac{1}{(x-1)}$ C $y = \dfrac{1}{(x-2)}$ D $y = \dfrac{-1}{(x-1)}$ E $y = \dfrac{-1}{(x-2)}$

22. The diagram shows two overlapping triangles; an isosceles triangle with an angle of 120° and an equilateral triangle with area 36. Two of the vertices of the equilateral triangle are midpoints of the equal sides of the isosceles triangle.



What is the total area of the shaded regions (inside the isosceles triangle but outside the equilateral triangle)?

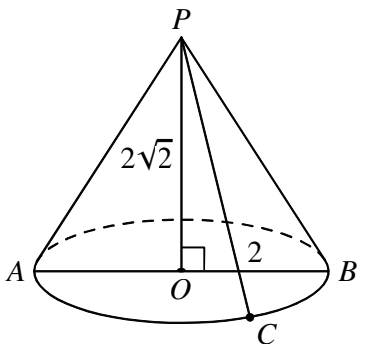
A 24 B 26 C 28 D 30 E 32

23. For particular real numbers a and b, the function f is defined by $f(x) = ax + b$, and is such that $f\big(f\big(f(x)\big)\big) = 27x - 52$.

Which of the following formulas defines the function g such that, for all values of x, $g\big(f(x)\big) = x$?

A $\frac{1}{3}x - 4$ B $\frac{1}{3}x + \frac{4}{3}$ C $4x - 3$ D $\frac{1}{3}x - \frac{4}{3}$ E $3x - 4$

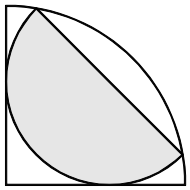
24. The diagram shows a circle with centre O which lies in a horizontal plane. The diameter AB has length 4. Point P lies vertically above O and $PO = 2\sqrt{2}$. Point C lies on the semicircular arc AB such that the ratio of the lengths of the arcs AC and CB is 2 : 1.



What is the shortest distance from A to PC?

A $\sqrt{2}$ B $\sqrt{3}$ C 2 D $2\sqrt{2}$ E 3

25. A semicircle is inscribed in a quarter circle as shown.



What fraction of the quarter circle is shaded?

A $\frac{1}{3}$ B $\frac{1}{\sqrt{3}}$ C $\frac{2}{3}$ D $\frac{\sqrt{3}}{2}$ E $\frac{1}{\sqrt{2}}$

