United Kingdom
Mathematics Trust

Andrew Jobbings Senior Kangaroo

Wednesday 16 November 2022

a member of the Association Kangourou sans Frontières

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England & Wales: Year 13 or below
Scotland: S6 or below
Northern Ireland: Year 14 or below

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil** to record your answer to each problem as a three-digit number from 000 to 999.
   Pay close attention to the example on the Answer Sheet that shows how to code your answers.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer;
   There is no penalty for giving an incorrect answer.
7. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Andrew Jobbings Senior Kangaroo should be sent to:

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1. The number x satisfies $x^2 - 4x + 2 = 0$. What is the value of $x + \frac{2}{x}$?

2. The sum of the ages of Tom and John is 23, the sum of the ages of John and Alex is 24 and the sum of the ages of Tom and Alex is 25. What is the sum of the squares of their three ages?

3. Zia conducted a study of the number of coins each member of her club had in their pocket. To her surprise, nobody had zero coins and exactly seven people had only one coin each. Everybody else had at least two coins. She drew a pie chart of her results, and to her surprise found each sector had an angle which was an integer ten degrees larger than its immediate predecessor. The smallest sector in the pie chart had an angle of 35^o. How many people are in Zia's club?

4. The rectangles S_1 and S_2 are drawn on a set of coordinate axes. S_1 has vertices at $(0, y)$; (x, y); $(x, 4)$; and $(0, 4)$, where $x < 9$ and $y < 4$. S_2 has vertices at $(x, 0)$; $(9, 0)$; $(9, y)$; and (x, y). The rectangles are equal in area. What is the value of $\frac{360x}{y}$?

5. Paul is painting a wall. He knows the area of the wall is 1920 square metres, correct to the nearest ten. He uses tins of paint, each of which can cover 18 square metres, correct to the nearest integer.
He needs to paint the wall completely, and still have at least half a tin of paint left over for any minor repairs.
What is the smallest number of tins he must buy to be sure of having enough paint?

6. A cube is dissected into 6 pyramids by connecting a given point in the interior of the cube with each vertex of the cube, so that each face of the cube forms the base of a pyramid. The volumes of five of these pyramids are 200, 500, 1000, 1100 and 1400. What is the volume of the sixth pyramid?

7. The positive integer N has exactly six distinct (positive) factors including 1 and N. The product of five of these is 6075. What is the value of the sixth factor?

8. In a right-angled triangle ABC (with right angle at A) the bisectors of the acute angles intersect at point P. The distance from P to the hypotenuse is $\sqrt{80000}$. What is the distance from P to A?

9. Using the 24-hour clock, the time 16:11:22 on the date 16/11/22 (i.e. the 16th of November 2022) has hours, minutes and seconds equal to date, month and (two-digit) year respectively. Let S be the number of seconds which will elapse after that date and time until this phenomenon next occurs.
What is the value of $\sqrt{S}$?

10. The equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ both have real roots. It is known that the sum of the squares of the roots of the first equation is equal to the sum of the squares of the roots of the second one, and that $a \neq b$. Find the value of $500 + 10(a + b)$.

11. A quadrilateral contains an inscribed circle (i.e. a circle tangent to the four sides of the quadrilateral). The ratio of the perimeter of the quadrilateral to that of the circle is $4 : 3$. The ratio of the area of the quadrilateral to that of the circle, expressed in its lowest terms, is $a : b$. What is the value of $100a + 49b$?

12. Five cards have the numbers 101, 102, 103, 104 and 105 on their fronts.

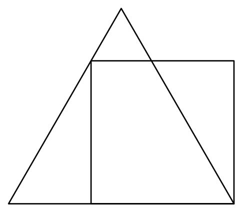
101	102	103	104	105

On the reverse, each card has one of five different positive integers: a, b, c, d and e respectively. We know that $a + 2 = b - 2 = 2c = \frac{d}{2} = e^2$.
Gina picks up the card which has the largest integer on its reverse. What number is on the front of Gina's card?

13. A sequence is given by $x_1 = 2$, $x_{n+1} = \frac{{x_n}^{x_n}}{32^{n-1}}$. What is $\frac{x_4}{128}$?

14. A class consists of pupils who each wear either a red tie or a green tie. There are two more pupils wearing green ties than wearing red ties. Two pupils are chosen at random. The probability of selecting two pupils of different tie-colour is exactly treble the probability of selecting two pupils who both wear red ties. Given that R is the number of pupils wearing a red tie, and that $R > 0$, determine $R^3 + R^2 + R$.

15. The perimeter of the square in the figure is 40. The perimeter of the larger equilateral triangle in the figure is $a + b\sqrt{p}$, where p is a prime number. What is the value of $7a + 5b + 3p$?



16. On the island of Friends and Foes, every citizen is either a Friend (who always tells the truth) or a Foe (who always lies). Seven citizens are sitting in a circle. Each declares "I am sitting between two Foes". How many Friends are there in the circle?

17. What is the smallest three-digit positive integer which can be written in the form pq^2r, where p, q and r are distinct primes?

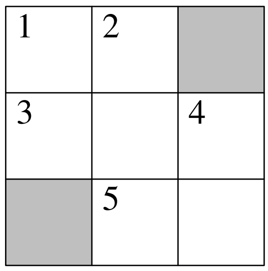
18. The number $\dfrac{20! \times 22!}{16! \times 11!}$ has N prime factors, which are not necessarily distinct.

What is the value of $N(N - 2)$?

19. How many different real number solutions are there to the following equation?

$$\left(x^2 - 8x + 15\right)^{\left(x^5 - 5x^3 + 4x\right)} = 1$$

20. Each cell in this cross-number can be filled with a non-zero digit such that all of the conditions in the clues are satisfied. The digits used are not necessarily distinct.
Determine T, the sum of all six answers to the clues. The answer to this question is $0.5 \times T$.



ACROSS:
1. Two less than a prime.
3. Exactly 100 more than its largest proper factor.
5. A multiple of 13.

DOWN:
1: A fourth power.
2: A cube.
4: Not prime, not square, not even.

