UK Junior Mathematical Olympiad 2017

Organised by The United Kingdom Mathematics Trust

Tuesday 13th June 2017

RULES AND GUIDELINES :
READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, measuring instruments and squared paper is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. **Write in blue or black pen or pencil.**
   For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work.
   For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.
   Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.
   Do not hand in rough work.
5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first 30 minutes so as to allow well over an hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*.
   This means that each answer must be accompanied by clear explanations and proofs.
   Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you are not able to do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

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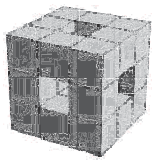
Section A

Try to complete Section A within 30 minutes or so. Only answers are required.

A1. How many centimetres are there in 1 km 2 m 3 cm 4 mm?

A2. The solid shown is formed by taking a 3 cm × 3 cm × 3 cm cube and drilling a 1 cm × 1 cm square hole from the centre of each face to the centre of the opposite face.



What is the volume in cm^3 of the solid?

A3. Howard is out running. He is now $\frac{3}{5}$ of the way through the second half of his run. What fraction of the whole run has he completed?

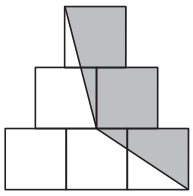
A4. A bookmark-maker sells bookmarks for £1 each or 7 for £6. What is the smallest amount you could pay for 2017 of her bookmarks?

A5. In 1866, the yacht *Henrietta* – with Gordon Bennett aboard – won the Great Ocean Yacht Race, travelling a distance of approximately 3000 nautical miles. The winning time was 13 days and 22 hours, to the nearest hour.

What was the yacht's average speed in nautical miles per hour, to the nearest integer?

A6. The diagram shows six identical squares arranged symmetrically.

What fraction of the diagram is shaded?



A7. A fully-grown long-tailed tit − *Aegithalos caudatus* − weighs only 9 g, whereas a £1 coin weighs 9.5 g.

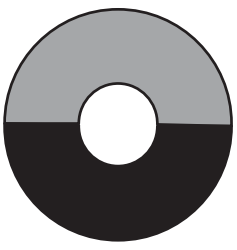
To the nearest 1 %, what percentage of the weight of a £1 coin is the weight of a fully-grown long-tailed tit?

A8. A jar contains red and white marbles in the ratio 1 : 4. When Jenny replaces 2 of the white marbles with 7 red marbles, the ratio becomes 2 : 3.

What is the ratio of the total number of marbles in the jar now to the total number in the jar before?

A9. How many multiples of 3 that are less than 1000 are not divisible either by 9 or by 10?

A10. Two concentric circles are drawn, as shown in the diagram. Concentric circles share the same point as their centre. The radius of the smaller circle is a third of the radius of the larger circle. The top half of the larger circle which is outside the smaller circle, is shaded in grey.



The ratio of the grey shaded area to the area of the smaller circle in its simplest form is $a : b$. What are the values of a and b?

Section B

Your solutions to Section B will have a major effect on your JMO result. Concentrate on one or two questions first and then **write out full solutions** (not just brief 'answers').

B1. An amount of money is to be divided equally between a group of children. If there was 20p more than this amount, then there would be enough for each child to receive 70p. However, if each child was to receive 60p, then £2.10 would be left over.

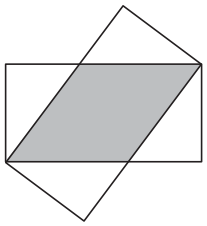
How many children are there in the group?

B2. A 3-digit integer is called a 'V-number' if the digits go 'high-low-high' – that is, if the tens digit is smaller than both the hundreds digit and the units (or 'ones') digit.

How many 3-digit 'V-numbers' are there?

B3. Two identical rectangles overlap in such a way that a rhombus is formed, as indicated in the diagram. The area of the rhombus is five-eighths of the area of each rectangle.

What is the ratio of the length of the longer side of the rectangle to the length of the shorter side?



B4. My uncle lives a long way away and his letters always contain puzzles. His three local teams are the Ants (A), the Bees (B), and the Cats (C), who play each other once a year.

My uncle claimed that the league table part way through the year looked like this:

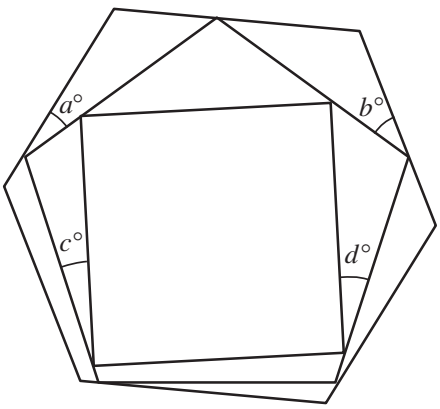
	Played	Won	Drawn	Lost	Goals for	Goals against
A	1	0	0	1	4	2
B	2	1	1	0	2	2
C	2	1	0	1	3	1

When we complained that this is impossible, he admitted that every single number was wrong but he excused himself because every number was exactly '1 out'.

Find the correct table, explaining clearly how you deduced the corrections.

B5. The diagram shows a square whose vertices touch the sides of a regular pentagon. Each vertex of the pentagon touches a side of a regular hexagon.

Find the value of $a + b + c + d$.



B6. The 9-digit positive integer N with digit pattern $ABCABCBBB$ is divisible by every integer from 1 to 17 inclusive.

The digits A, B and C are distinct. What are the values of A, B and C?