



UK Maths Trust

Mentoring Scheme

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Sample questions

These questions are taken from the first sheet and give a good indication of the level of difficulty and prerequisite knowledge required at the start of the programme.

1. Does there exist a square number whose digit sum is 123? Justify your answer.
2. The collection $\{8, 9, 12\}$ is a set of three positive integers with the property that, given any two of these integers, their greatest common divisor (gcd) is equal to their difference:
 - $\gcd(8, 9) = 1 = 9 - 8$;
 - $\gcd(8, 12) = 4 = 12 - 8$;
 - $\gcd(9, 12) = 3 = 12 - 9$.

Can you find a set of four integers with the same property? What about five? Or six?

3. The point P lies on the circumcircle of the triangle ABC . Perpendiculars PL , PM and PN are drawn to the sides BC , CA and AB respectively. If necessary, the sides of the triangle are produced (extended). Prove that the points L , M and N are collinear (lie on the same straight line).