



UK Maths Trust

Mentoring Scheme

Srinivasa Ramanujan

Sample questions

These questions are taken from the first sheet and give a good indication of the level of difficulty and prerequisite knowledge required at the start of the programme.

1. Let ABC be a triangle and O be the centre of the circle passing through A , B and C (which has radius R). Define the vectors $\mathbf{a} = \overrightarrow{OA}$, $\mathbf{b} = \overrightarrow{OB}$ and $\mathbf{c} = \overrightarrow{OC}$.
 - a) Write down expressions for $\mathbf{a} \cdot \mathbf{a}$ and $\mathbf{a} \cdot \mathbf{b}$ in terms of R and the angles of the triangle.
 - b) Let H be the point such that $\overrightarrow{OH} = \mathbf{a} + \mathbf{b} + \mathbf{c}$. Show that H lies on all three altitudes of the triangle (H is the *orthocentre* of the triangle). *Note: an altitude of a triangle is a line passing through a vertex which is perpendicular to the opposite side.*
 - c) Show that the three medians pass through a point, which we will denote by G (the *centroid* of the triangle). Further show that O , G and H are collinear and determine $\frac{OH}{OG}$. *Note: a median of a triangle is a line passing through a vertex and the midpoint of the opposite side.*
 - d) Show that there is a point N lying on the line OH , which is the centre of a circle passing through all three midpoints of the sides as well as the midpoints of segments AH , BH and CH .
 - e) Further show that circle in part (d) passes through the feet of the three altitudes. This circle is called the *nine-point circle*.

2. You may have met Ms X MP and her intern Jane in G H Hardy, Sheet 8, Question 6; now Ms X has a new project for her assistant. “I had a very good socio-economic model to develop employment opportunities in my constituency. Unfortunately your predecessor did not collect all the relevant data and time was wasted.” (Jane prudently keeps quiet.) Ms X goes on to explain that the model requires the solution of the following system of equations.

$$a + b = 18 \quad (1)$$

$$au + bv = 110 \quad (2)$$

$$au^2 + bv^2 = 690 \quad (3)$$

$$au^3 + bv^3 = 4430 \quad (4)$$

She adds that, when she was at school, she met only systems of linear simultaneous equations. Show how Jane can again reassure Ms X by solving the equations.

3. On the same axes, sketch the graphs

$$y = (x + 1)^3 - (x + 1) \quad \text{and} \quad y^2 = (x + 1)^3 - (x + 1).$$

You should justify all the features of the graphs that you show and give the co-ordinates of any significant points. In particular, give details of the points where the two graphs intersect.