UK Maths Trust

Andrew Jobbings Senior Kangaroo

Wednesday 15 November 2023

© 2023 UK Mathematics Trust

a member of the Association Kangourou sans Frontières

supported by Overleaf

England & Wales: Year 13 or below
Scotland: S6 or below
Northern Ireland: Year 14 or below

Instructions

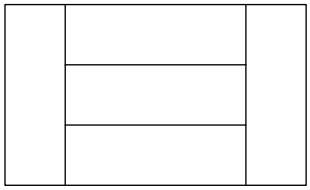
1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil** to record your answer to each problem as a three-digit number from 000 to 999.
   Pay close attention to the example on the Answer Sheet that shows how to code your answers.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer;
   There is no penalty for giving an incorrect answer.
7. **Your Answer Sheet will be read by a machine.** Do not write or doodle on the sheet except to mark your chosen options. The machine will read all black pencil markings even ifthey are in the wrong places. If you mark the sheet in the wrong place, doodle, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way, or reject the answer sheet.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Andrew Jobbings Senior Kangaroo should be sent to:

challenges@ukmt.org.uk

www.ukmt.org.uk

1. A particular flag is in the shape of a rectangle divided into five smaller congruent rectangles as shown. When written in its lowest terms, the ratio of the side lengths of the smaller rectangle is $\lambda : 1$, where $\lambda < 1$.
What is the value of 360λ?



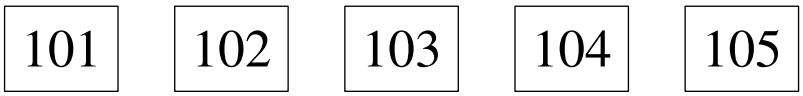
2. The integers m and n satisfy the equation $3^m \times n = 7! + 8! + 9!$.
What is the smallest possible value for n?

3. How many of the integers from 2^{10} to 2^{18} inclusive are divisible by 2^9?

4. Months of the year are usually labelled numerically by '01' for January, '02' for February, and so on, through to '12' for December. Lydia notices that during January, the number of letters in the name of the month is greater than the month's numerical label (i.e. $7 > 1$).
For how many days during 2024 will the date have that property?

5. The number C is defined as the sum of all the positive integers n such that $n - 6$ is the second largest factor of n. What is the value of $11C$?

6. Five cards have the numbers 101, 102, 103, 104 and 105 on their fronts.



On the reverse, each card has a statement printed as follows:
101: The statement on card 102 is false
102: Exactly two of these cards have true statements
103: Four of these cards have false statements
104: The statement on card 101 is false
105: The statements on cards 102 and 104 are both false
What is the total of the numbers shown on the front of the cards with TRUE statements?

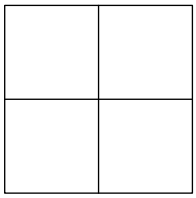
7. A container in the shape of a rectangular box is partially filled with $120\,\text{m}^3$ of water. The depth of the water is either 2 m, 3 m or 5 m, depending on which side of the box is on the ground.
What is the volume of the container in m^3?

8. The product of all the factors of 5^{15} is 5^P.
What is the value of P?

© 2023 UK Mathematics Trust

9. What is the lowest prime number that is thirteen more than a cube?

10. The smallest four two-digit primes are written in different squares of a 2×2 table.
The sums of the numbers in each row and column are calculated.
Two of these sums are 24 and 28.
The other two sums are c and d, where $c < d$.
What is the value of $5c + 7d$?



11. The town of Coppersville uses some special coins. Edward has five 5 p coins, four 4 p coins, three 3 p coins, two 2 p coins and one 1 p coin. The smallest amount he cannot pay with his coins alone is W p.
What is the value of W?

12. How many positive integers are there whose digits do not include 0, and whose digits have sum 6?

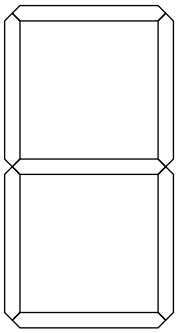
13. On a square $ABCD$ a line segment BE is drawn such that the point E lies on the side CD.
The perimeter of triangle BCE is three-quarters of the perimeter of the square $ABCD$.
The ratio of lengths $CE : CD$ is $\lambda : 1$.
What is the value of $960 \times \lambda$?

14. Three different numbers are chosen at random from the list 1, 3, 5, 7, 9, 11, 13, 15, 17, 19.
The probability that one of them is the mean of the other two is p.
What is the value of $\frac{120}{p}$?

15. Kiran is designing a game which involves a bag of twenty-one marbles. Some of the marbles are blue, the rest are red. To play the game, two marbles are drawn out. The game is won if at least one red marble is drawn. To ensure the probability of the game being won is exactly one-half, Kiran uses B blue marbles and R red marbles.
What is the value of $B^2 + R^2$?

© 2023 UK Mathematics Trust

16. Stephen's calculator displays only one digit, as shown in the diagram. Unfortunately, the calculator is broken. Each time he switches it on, each of the seven bars will either illuminate (show up) or not, with probability 0.5. The resultant display correctly shows one of the ten digits 0 - 9 with probability $\frac{a}{b}$.
Given that $\frac{a}{b}$ is written in its lowest terms, what is the value of $9a + 2b$?

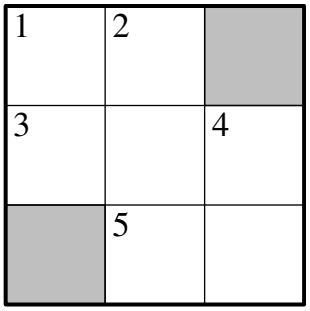


17. The integer part of $\sqrt{72 + \sqrt{72 + \sqrt{72 + \sqrt{72 + \sqrt{72 + \sqrt{72}}}}}}$ is A.
What is the value of $50A$?

18. The number 2024 may be split into its first two digits and its last two digits to form the numbers 20 and 24. The highest common factor of these numbers, HCF(20,24) is equal to 4. Similarly, 2025 may be split, and HCF(20,25)=5.
For how many remaining years this century (i.e. after 2025 and up to and including 2099) will this highest common factor be equal to one?

19. Next year's 'leap day', 29th February 2024, may be written in 'ddmmyy' format as the number 290224. What is the sum of all the two-digit factors of 290224?

20. Each cell in this cross-number can be filled with a non-zero digit so that all of the conditions in the clues are satisfied.



The digits used are not necessarily distinct.

ACROSS
1. Four less than a factor of 105.
3. One more than a palindrome.
5. The square-root of the answer to this Kangaroo question.

DOWN
1. Two less than a square.
2. Four hundred less than a cube.
4. Six less than the sum of the answers to two of the other clues.

What is the square of the answer to 5 ACROSS?

© 2023 UK Mathematics Trust