



**UK Maths Trust**

## **Mentoring Scheme**

Supported by **[XTX]**  
MARKETS

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### **Sample questions**

These questions are taken from the first sheet and give a good indication of the level of difficulty and prerequisite knowledge required at the start of the programme.

1. How many positive integers less than or equal to 1000 are not divisible by any of 2, 3 or 5?
2. Will, Xavier, Yolanda and Zara are sixth-formers who like to work on maths problems together. Sometimes they tease their teacher by handing in a single, typed set of solutions. When questioned on one occasion, they make the following set of statements, each of which is either true or false.

Will: It was Xavier.  
Xavier: It was Zara.  
Yolanda: It was not me.  
Zara: What Xavier says is wrong.

- a) Let us suppose that exactly one of the four is lying and the other three are telling the truth. Who is lying? Who typed out the solutions?
  - b) Let us now suppose that exactly one of the four is telling the truth and the other three are lying. Who is telling the truth? Who typed out the solutions?
  - c) Now suppose that two of the statements are true and two are false. Who are the people who could have typed out the solutions?
  - d) Without prior knowledge of the number of true statements, what can we say about the identity of the typist?
3. Prove that for all positive real numbers  $a, b, c$ :
    - a)  $(a + b)^2 \geq 4ab$ ;
    - b)  $\frac{a+b+c}{2} \geq \frac{bc}{b+c} + \frac{ca}{c+a} + \frac{ab}{a+b}$ .