UK Maths Trust

Mentoring Scheme

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Sample questions

These questions are taken from the first sheet and give a good indication of the level of difficulty and prerequisite knowledge required at the start of the programme.

1. Does there exist a square number whose digit sum is 123? Justify your answer.

2. The collection $\{8, 9, 12\}$ is a set of three positive integers with the property that, given any two of these integers, their greatest common divisor (gcd) is equal to their difference:

- $\gcd(8, 9) = 1 = 9 - 8$;
- $\gcd(8, 12) = 4 = 12 - 8$;
- $\gcd(9, 12) = 3 = 12 - 9$.

Can you find a set of four integers with the same property? What about five? Or six?

3. The point P lies on the circumcircle of the triangle ABC. Perpendiculars PL, PM and PN are drawn to the sides BC, CA and AB respectively. If necessary, the sides of the triangle are produced (extended). Prove that the points L, M and N are collinear (lie on the same straight line).