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Grey Kangaroo

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

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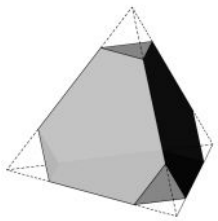
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
D	D	B	C	C	C	B	A	A	D	E	D	B	E	E	A	C	E	D	D	E	D	B	C	A

1. Julio cuts off the four corners, or vertices, of a regular tetrahedron, as shown.
How many vertices does the remaining shape have?



A 8 B 9 C 11 D 12 E 15

SOLUTION **D**

A tetrahedron has four corners (or vertices). When a corner is removed, three new vertices are formed. Therefore the number of vertices the remaining shape will have is $4 \times 3 = 12$.

2. What is the value of $\dfrac{20 \times 24}{2 \times 0 + 2 \times 4}$?

A 12 B 30 C 48 D 60 E 120

SOLUTION **D**

Simplifying the expression gives $\dfrac{20 \times 24}{2 \times 0 + 2 \times 4} = \dfrac{20 \times 24}{0 + 8} = \dfrac{20 \times 24}{8} = 20 \times 3 = 60$.

3. Ria has three counters marked 1, 5 and 11, as shown. She wants to place them side-by-side to make a four-digit number. How many different four-digit numbers can she make?

A 3 B 4 C 6 D 8 E 9

SOLUTION **B**

The three counters can be arranged in six ways. However, when the counters with 1 and 11 on are placed side-by-side, the four-digit number formed will look the same whichever way round these two counters are placed. As these counters can be next to each other if they are both before the counter with five on or both after the counter with 5 on, this means only four different numbers can be made, namely 1115, 1151, 1511 and 5111.

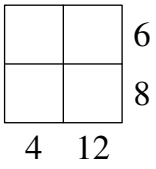
4. The maximum weight allowed in a service lift is satisfied exactly by 12 identical large packages or by 20 identical small packages. What is the greatest number of small packages that the lift can carry alongside nine large packages?

A 3 B 4 C 5 D 6 E 8

SOLUTION **C**

Since the lift can take 12 large packages or 20 small packages, every three large packages can be replaced by five small packages. Therefore, the greatest number of small packages that can be carried with nine (= 12 − 3) large packages is 5.

5. Four different positive integers are placed on a grid and then covered up. The product of the integers in each row and column is shown in the diagram.
What is the sum of the four integers?



A 10 B 12 C 13 D 14 E 15

SOLUTION **C**

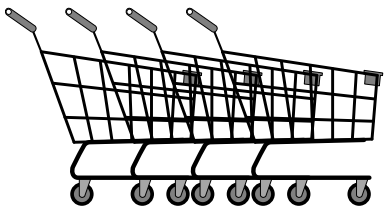
Let the four integers in the grid be p, q, r and s, as shown here. Since the four integers are different and the product of the integers in the first column is 4, we have p and r as 1 and 4 in some order. However, we also know that $p \times q = 6$ and hence p cannot be 4. Therefore $p = 1$ and hence $q = 6$, $r = 4$ and $s = 2$.

p	q
r	s

Therefore the sum of the four integers in the grid is $1 + 6 + 4 + 2 = 13$ and the completed grid is as shown below.

1	6
4	2

6. The length of a set of four well-parked and fitted trolleys is 108 cm. The length of a set of ten well-parked and fitted trolleys is 168 cm.
What is the length of a single trolley?



A 60 cm B 68 cm C 78 cm D 88 cm E 90 cm

SOLUTION **C**

Six extra trolleys added 60 cm to the length of the set. Therefore, each extra trolley adds $60\,\text{cm} \div 6 = 10\,\text{cm}$ to the set. Since a set of four trolleys has length 108 cm, the length of a single trolley is $(108 - 3 \times 10)$ cm, or 78 cm.

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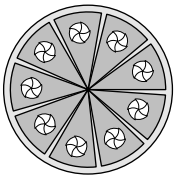
7. Carina baked a cake and cut it into ten equal pieces. She ate one piece and then arranged the remaining pieces evenly, as shown. What is the size of the angle at the centre of the cake between any two adjacent pieces?

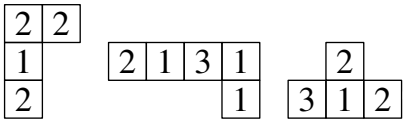


A 5° B 4° C 3° D 2° E 1°

SOLUTION **B**

When the cake is cut into 10 equal pieces, the size of the angle at the centre of each piece is $360° \div 10 = 36°$. Therefore, when one piece is removed and the pieces rearranged, the sum of the nine equal angles created at the centre of the cake is 36°. Hence the size of each angle between two adjacent pieces is $36° \div 9 = 4°$.

8. Werner can make a 4×4 square, where the sums of the numbers in all four rows and in all four columns are the same, from the three pieces shown and one further piece. Which of the following pieces is needed to complete his square?



A 1 1 3 B 2 1 0 C 1 2 1 D 2 2 2 E 2 2 3

SOLUTION **A**

The second piece in the question contains a row of four cells with sum $2 + 1 + 3 + 1 = 7$. Therefore the total of all the numbers in the square is $4 \times 7 = 28$. The sum of the numbers on the three pieces shown is $7 + 8 + 8 = 23$. Therefore the sum of the numbers on the missing piece is $28 - 23 = 5$. The only one of the pieces in the options with a sum of 5 is A. Therefore the missing piece is piece A and the completed square is as shown below.

2	1	3	1
2	2	2	1
1	3	1	2
2	1	1	3

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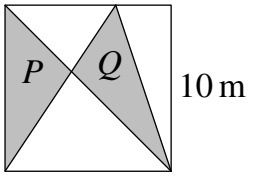
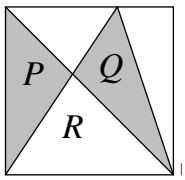
9. A square has side-length 10 m. It is divided into parts by three straight line segments, as shown. The areas of the two shaded triangles are $P\,\text{m}^2$ and $Q\,\text{m}^2$.
What is the value of $P - Q$?



A 0 B 1 C 2 D 5 E 10

SOLUTION **A**

Let $R\,\text{m}^2$ be the area of the triangle shown on the diagram. The triangle with area $(P + R)\,\text{m}^2$ has base 10 m and height 10 m. Similarly, the triangle with area $(Q + R)\,\text{m}^2$ also has base 10 m and height 10 m. Therefore the areas of these two triangles are equal and hence $P + R = Q + R$. Therefore $P = Q$ and hence the value of $P - Q$ is 0.



10. Paula the penguin goes fishing every day and always brings back twelve fish for her two chicks.
Each day, she gives the first chick she sees seven fish and gives the second chick five fish, which they eat. In the last few days one chick has eaten 44 fish. How many has the other chick eaten?

A 34 B 40 C 46 D 52 E 64

SOLUTION **D**

The first chick has eaten 44 fish in groups of seven and five. When you subtract multiples of five repeatedly from 44, the only multiple of seven that is obtained is 14. Therefore the first chick has eaten seven fish twice and five fish six times. Therefore, the number of times Paula went fishing is $2 + 6 = 8$ and so the number of fish she caught is $8 \times 12 = 96$. Therefore the number of fish the second chick has eaten in that time is $96 - 44 = 52$.

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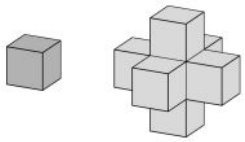
11. Johan has a large number of identical cubes. He has made the structure on the right by taking a single cube and then sticking another cube to each face. He wants to make an extended structure in the same way so that each face of the structure on the right will have a cube stuck to it. How many extra cubes will he need to complete his extended structure?



A 10 B 12 C 14 D 16 E 18

SOLUTION **E**

In the given structure, there are five exposed faces on each of the six cubes that were added to the original cube and hence there are 30 faces to cover on the structure. Adding a cube so that one of its edges touches an edge of the original cube covers two faces on the structure. The original cube has 12 edges and so 12 cubes are required to cover 24 of the faces of the structure. A further six cubes are required to cover the final six outer faces of the structure. Therefore the number of extra cubes Johan needs to complete his extended structure is $12 + 6 = 18$.

12. Kenny the kangaroo jumps up a mountain and then jumps back down along the same route. He covers three times the distance with each downhill jump as he does with each uphill jump. Going uphill, he covers 1 metre per jump. In total, Kenny makes 2024 jumps. What is the total distance, in metres, that Kenny jumps?

A 506 B 1012 C 2024 D 3036 E 4048

SOLUTION **D**

Since Kenny covers three times the distance with each downhill jump as he does with each uphill jump, he will make three times as many jumps going uphill as he does covering the same distance going downhill. The question tells us that Kenny makes 2024 jumps in total and so $\frac{3}{4}$ of these, namely 1518 jumps, will be uphill. Now, since Kenny covers 1 metre per uphill jump, the distance he jumps uphill is 1518 metres. Therefore the total distance Kenny jumps is 2×1518 metres, or 3036 metres.

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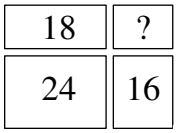
13. Gerard cuts a large rectangle into four smaller rectangles. The perimeters of three of these smaller rectangles are 16, 18 and 24, as shown in the diagram.
What is the perimeter of the fourth small rectangle?

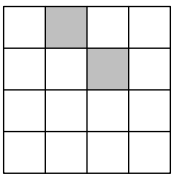


A 8 B 10 C 12 D 14 E 16

SOLUTION **B**

Consider the two rectangles with perimeter 18 and 16. It can be seen in the diagram that the sum of the lengths of horizontal sides of these rectangles is equal to the sum of the lengths of the horizontal sides of the large rectangle. Similarly the sum of the lengths of the vertical sides of these two rectangles is equal to the lengths of the vertical sides of the large rectangle. Hence the perimeter of the large rectangle is equal to the sum of the perimeters of these two rectangles. A similar argument can be applied to the rectangle with perimeter 24 and the rectangle with the unknown perimeter. Therefore $18 + 16 = 24 + P$, where P is the unknown perimeter. Hence the perimeter of the fourth small rectangle is 10.

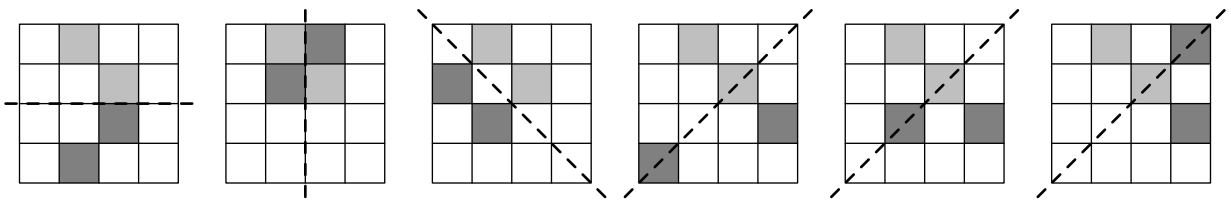
14. Tarek wants to shade two further squares on the diagram shown so that the resulting pattern has a single axis of symmetry. In how many different ways can he complete his pattern?



A 2 B 3 C 4 D 5 E 6

SOLUTION **E**

There are four possible axes of symmetry for the large square: one horizontal, one vertical, one diagonal from top left to bottom right and one diagonal from top right to bottom left. For the first three cases, there is only one way to shade two further squares to complete a symmetric pattern, as shown in the first three diagrams below. However, for the axis of symmetry running from top right to bottom left, while one particular square must be shaded to complete a symmetric pattern, the second square could be chosen to be any one of the three remaining squares on the axis of symmetry, as shown in the final three diagrams. Hence there are six different ways in which Tarek can complete his pattern.



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15. Nine cards numbered from 1 to 9 were placed facedown on the table. Aleksa, Bart, Clara and Deindra each picked up two of the cards. Aleksa said, "My numbers add up to 6." Bart said, "The difference between my numbers is 5." Clara said, "The product of my numbers is 18." Deindra said, "One of my numbers is twice the other one." All four made a true statement. Which number was left on the table?

A 1 B 3 C 6 D 8 E 9

SOLUTION **E**

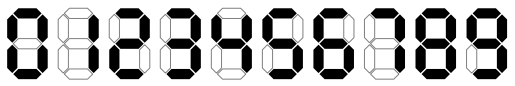
Since all four made a true statement, the numbers on Aleksa's cards were 1 and 5 or 2 and 4; the numbers on Bart's cards were 1 and 6, 2 and 7, 3 and 8 or 4 and 9; the numbers on Clara's cards were 2 and 9 or 3 and 6 and the numbers on Deindra's cards were 1 and 2, 2 and 4, 3 and 6 or 4 and 8.

Suppose the numbers on Aleksa's cards were 2 and 4. Then the numbers on Clara's cards could only be 3 and 6. However, this would not leave any possible pair of cards for Deindra. Therefore the numbers on Aleksa's cards were 1 and 5.

Now suppose the numbers on Clara's cards were 2 and 9. Then the numbers on Bart's cards could only be 3 and 8. However, this would again leave no possible pair of cards for Deindra. Therefore the numbers on Clara's cards were 3 and 6.

Finally, suppose the numbers on Bart's cards were 4 and 9. Again this would leave no possible pair of cards for Deindra. Therefore the numbers on Bart's cards were 2 and 7, leaving the cards numbered 4 and 8 for Deindra. Therefore the number left on the table was 9.

16. The digits 0 to 9 can be drawn with horizontal and vertical segments, as shown. Greg chooses three different digits. In total, his digits have 5 horizontal segments and 10 vertical segments. What is the sum of his three digits?



A 9 B 10 C 14 D 18 E 19

SOLUTION **A**

The number of horizontal segments in Greg's three digits is five. Since the maximum number of horizontal segments in any digit is three, these five segments could be arranged as 0, 2 and 3 or 1, 1 and 3 or 1, 2 and 2. However, only the digit 0 has two horizontal segments so an arrangement using three different digits with 1, 2 and 2 horizontal segments is not possible. There are two different digits with one horizontal segment, namely 4 and 7, but these only contain a total of five vertical segments leaving the third digit needing five vertical segments when only a maximum of four is possible. Therefore the only possible arrangement of horizontal segments is 0, 2 and 3. Only the digit 1 has no horizontal segments and, as mentioned earlier, only the digit 0 has two horizontal segments. Between them the digits 1 and 0 have six vertical segments and hence the third digit has three horizontal segments and four vertical segments and only digit 8 satisfies these conditions. Therefore the sum of Greg's three digits is $0 + 1 + 8 = 9$.

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17. Water makes up 80 per cent of fresh mushrooms. However, water makes up only 20 per cent of dried mushrooms. By what percentage does the mass of a fresh mushroom decrease during drying?

A 60 B 70 C 75 D 80 E 85

SOLUTION **C**

Let the mass of a fresh mushroom be $20X$ grams. Since water makes up 80 per cent of this, the mass of water in the fresh mushroom is $16X$ grams and the mass of the remainder is $4X$ grams. In a dried mushroom, water makes up only 20 per cent. Therefore, the $4X$ grams of the remainder is 80 per cent of the dried mushroom and hence the mass of water in the dried mushroom is X grams. Therefore, the percentage decrease in the mass of a fresh mushroom during drying is $\dfrac{20X-(4X+X)}{20X}\times 100=\dfrac{15X}{20X}\times 100=75$.

18. How many eight-digit numbers can be written using only the digits 1, 2 and 3 so that the difference between any two adjacent digits is 1?

A 16 B 20 C 24 D 28 E 32

SOLUTION **E**

Since the difference between any two adjacent digits in the eight-digit number is 1, the number can contain no repeated digits and cannot have a 1 and a 3 next to each other. Therefore the structure of the eight-digit number is either $2p2q2r2s$ or $t2u2v2w2$, where p, q, r, s, t, u, v and w are either 1 or 3. Hence the total number of such eight-digit numbers is $2\times(2\times2\times2\times2)=32$.

19. A group of 50 girls sit in a circle. They throw a ball around the circle. Each girl who gets the ball throws it to the girl sitting six places anticlockwise from her, who catches it. Freda catches the ball 100 times. In that time, how many girls never get to catch the ball?

A 0 B 8 C 10 D 25 E 40

SOLUTION **D**

For Freda to catch the ball a second time, the ball must pass a multiple of 50 girls. Since the ball is thrown to the girl sitting six places anticlockwise from the thrower each time, the ball must also pass a multiple of 6 girls. The Lowest Common Multiple of 6 and 50 is 150 and hence the ball will have passed 150 girls before Freda catches the ball for a second time. In that time, the number of girls who have caught the ball is $150\div 6=25$. This pattern will then continue with the same 25 girls, including Freda, catching the ball before anyone catches it for a further time and so on. Therefore, the number of girls who never get to catch the ball is $50-25=25$.

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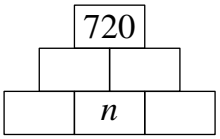
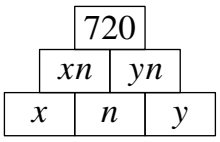
20. Donggyu wants to complete the diagram so that each box contains a positive integer and each box in the top two rows contains the product of the integers in the two boxes below it. He wants the integer in the top box to be 720. How many different values can the integer n take?



A 1 B 4 C 5 D 6 E 8

SOLUTION **D**

Let the integers in the left- and right-hand box of the bottom row be x and y respectively, as shown. Since each box in the second and third rows contain the product of the integers in the two boxes below them, the integers in the second row are xn and yn and the integer in the top row is xyn^2. The question tells us that this is to be equal to 720 and therefore n can be any integer such that its square is a factor of 720.



When 720 is written as a product of its prime factors, we obtain $720 = 2^4 \times 3^2 \times 5$. Therefore the possible values of n are $1, 2, 2^2, 3, 2 \times 3$ and $2^2 \times 3$. Hence there are six possible values of n.

21. Farmer Fi is selling chicken and duck eggs. She has baskets holding 4, 6, 12, 13, 22 and 29 eggs. Her first customer buys all the eggs in one basket. Fi notices that the number of chicken eggs she has left is twice the number of duck eggs. How many eggs did the customer buy?

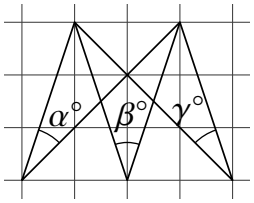
A 4 B 12 C 13 D 22 E 29

SOLUTION **E**

The total number of eggs in Farmer Fi's baskets initially is $4 + 6 + 12 + 13 + 22 + 29 = 86$. Once she sells one basket, she notices that the number of chicken eggs she has left is twice the number of duck eggs she has left. Therefore, the total number of eggs she has left is a multiple of 3. Hence the number of eggs that were bought by the first customer has the same remainder when divided by 3 as the original number of eggs. Since $86 = 3 \times 28 + 2$, that remainder is 2. Now 4, 13 and 22 all have a remainder of 1 when divided by 3 while both 6 and 12 are multiples of 3. However $29 = 3 \times 9 + 2$ and so has a remainder of 2 when divided by 3. Therefore the first customer bought 29 eggs.

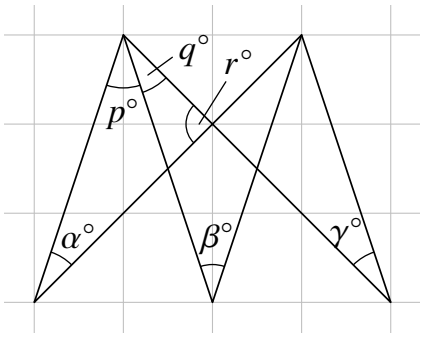
22. Three angles α°, β° and γ° are marked on squared paper, as shown.
What is the value of $\alpha + \beta + \gamma$?

A 60 B 70 C 75 D 90 E 120



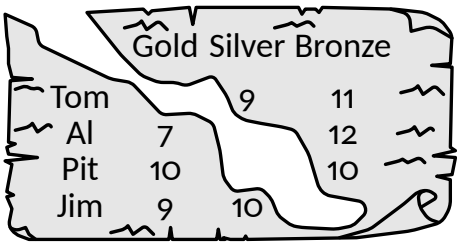
SOLUTION **D**

Let the three unknown angles in the triangle indicated be p°, q° and r°, as shown. Since the angle of size r° is made up of two 45° angles, we have $r = 90$. Also, since the triangle with p° at its vertex is congruent to the triangle with β° at its vertex, we have $p = \beta$. Using alternate angles with a pair of parallel lines, we also have $q = \gamma$. Since the angles in a triangle add to 180°, we have $\alpha + (p + q) + r = 180$. Therefore $\alpha + (\beta + \gamma) + 90 = 180$. Hence $\alpha + \beta + \gamma = 90$.



23. Captain Flint asked four of his pirates to write on a piece of paper how many gold, silver and bronze coins were in the treasure chest. Their responses are shown in the diagram but unfortunately part of the paper was damaged. Only one of the four pirates told the truth. The other three lied in all their answers. The total number of coins was 30. Who told the truth?

A Tom B Al C Pit D Jim
E We cannot be sure



SOLUTION **B**

If Tom told the truth, his answer for the number of gold coins would be 10. However, this is the same as Pit's answer and so Pit would have also told the truth for the number of gold coins. Therefore Tom could not have told the truth.
If Pit told the truth, his answer for the number of silver coins would be 10, which is the same as Jim's answer and so Jim would also have told the truth for the number of silver coins. Therefore Pit could not have told the truth.
If Jim told the truth, his answer for the number of bronze coins would be 11, which is the same as Tom's answer and so Tom would also have told the truth for the number of bronze coins. Therefore Jim could not have told the truth.
However, if Al told the truth, his answer for the number of silver coins would be 11, which does not match any other pirate's answer. His answers for the numbers of gold and bronze coins also do not match any other pirate's answer. Therefore Al told the truth.

24. Alex drives from point P to point Q, then immediately returns to P. Bob drives from point Q to point P, then immediately returns to Q. They travel on the same road, start at the same time and each travels at a constant speed. Alex's speed is three times Bob's speed. They meet each other for the first time 15 minutes after the start. How long after the start will they meet each other for the second time?

A 20 min B 25 min C 30 min D 35 min E 45 min

SOLUTION **C**

Let Bob's speed be V km/h. Therefore Alex's speed is $3V$ km/h. Since they meet each other for the first time after 15 minutes, or $\frac{1}{4}$ of an hour, the total distance between P and Q, in km, is $\frac{1}{4} \times V + \frac{1}{4} \times 3V = V$. Let the distance Bob has travelled when Alex passes him for the second time be Y km. Since time $= \dfrac{\text{distance}}{\text{speed}}$, we have $\dfrac{Y}{V} = \dfrac{V+Y}{3V}$ and hence $3Y = V + Y$, which has solution $Y = \dfrac{V}{2}$. Hence the time in hours that Bob has travelled when Alex passes him for the second time is $\dfrac{\frac{V}{2}}{V} = \dfrac{1}{2}$. Therefore they will meet for a second time after 30 minutes.

Note: When Alex passes Bob for the second time, they are both travelling in the same direction towards P, whereas when he passed Bob for the first time they were travelling in opposite directions.

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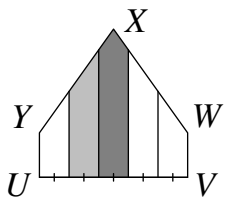
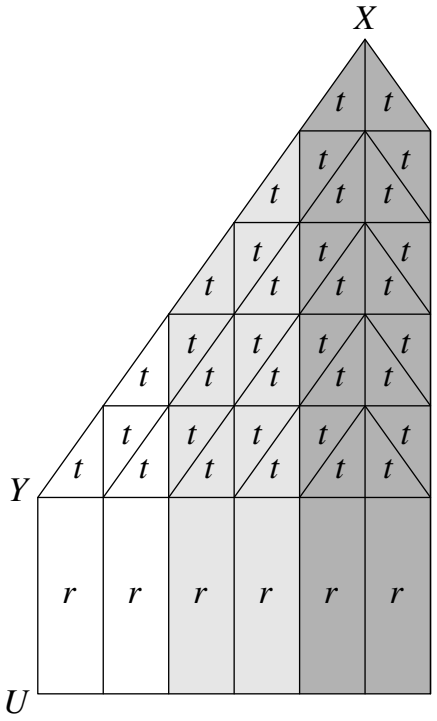
25. In the pentagon $UVWXY$, $\angle U = \angle V = 90°$, $UY = VW$ and $YX = XW$. Four equally spaced points are marked between U and V, and perpendiculars are drawn through each point, as shown in the diagram. The dark shaded region has an area of $13\,\text{cm}^2$ and the light shaded region has an area of $10\,\text{cm}^2$. What is the area, in cm^2, of the entire pentagon?



A 45 B 47 C 49 D 58 E 60

Solution **A**

Consider the left-hand part of the pentagon divided into congruent rectangles and congruent triangles, as shown. Let the area of each of the congruent rectangles be $r\,\text{cm}^2$ and let the area of each of the congruent triangles be $t\,\text{cm}^2$. Since the light shaded region is made up of two rectangles and twelve triangles and since the dark shaded region is made up of two rectangles and eighteen triangles, the information in the question tells us that $2r + 12t = 10$ and that $2r + 18t = 13$. Therefore $6t = 3$ and hence $t = \frac{1}{2}$. Then substitute back into the first equation to obtain $2r + 6 = 10$ and hence $2r = 4$ and then $r = 2$. Therefore the area of the whole pentagon, in cm^2, is $2 \times (r + t + r + 3t + r + 5t + r + 7t + r + 9t) = 2 \times (5r + 25t)$. Hence the total area, in cm^2, is $10r + 50t$ which, when you substitute in the values for r and t, gives $(20 + 25)\,\text{cm}^2$, or $45\,\text{cm}^2$.