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Cayley 2023 Markers' Report

Candidates in year 9 are not that used to what is expected from Olympiad-style problems. Despite this, there were many excellent solutions written up and it was impressive how many could cope with solving problems which are not standard compared with what they see in school. Some candidates don't quite understand the level of explanation that is needed. Many write waffling solutions, which explain trivial things in unnecessary detail, but others write too little and miss key points. The students should ask themselves whether the solution could be understood by someone who has never seen the correct solution and is following theirs and there would be no gaps in their thinking.
The paper proved to be quite accessible at all levels, with questions 1 and 2 very approachable for all, 3 and 4 a bit trickier and 5 and 6 more unusual, although there were more successful attempts at these than expected.
In questions where the idea is to find the best solution, many did not realise there are two aspects to this type of problem: proving that their solution works and that nothing better does.
There were some errors of basic arithmetic and students need to be confident working with numbers without relying on their calculators!

Q1

This accessible question was attempted by nearly all candidates, though only about half of the attempts were rigorous enough to warrant a high mark.
Most candidates attempted to use the divisibility by three and five rules to limit the number of possibilities, like the model solution. When doing so, it is essential that all cases are explicitly mentioned (five cases – one for each of the digits missing) and, ideally, the calculation accompanied by the conclusion is shown for each case.
Moreover, after eliminating the impossible cases, it remains necessary to check that all the remaining numbers are divisible by each of their respective digits.
A small number of candidates listed all the 120 numbers containing four different odd digits and claimed they had checked them all. While it is possible to do so, these checks (typically in form of division results and conclusions) for each number would have to be included to warrant a high mark for this approach.

Q2

This angle-chasing geometry problem was successfully solved by a majority of the candidates, making it the most approachable question on this year's Cayley paper.
Most candidates used one of the two approaches. One, like the model solution, based on expressing p and q in terms of $\angle EAD$ and $\angle ABD$. The other one based on labelling one of the unknown angles as x, and expressing one of the other unknown angles in two different ways in terms of x, p and q, resulting in an equation equivalent to $p = 2q$.
While most of the serious attempts at the question were successful, the quality of presentation of geometrical reasoning was varied.

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A formal proof must include steps in order of logical reasoning, ending with the conclusion $p = 2q$ in the final step. Each step of geometrical reasoning should be stated separately, including both the justification (e.g. points E, A, B are collinear) and the conclusion (e.g. $\angle CAB = 180–2a$).
Solutions that included all the steps in order but were vague on some of the justifications were treated comparatively leniently and deducted one or two marks. Those that did not attempt a structured argument or those that did not include any justification received very few marks.

Q3

This question turned out to be more challenging than it had initially appeared. With most candidates approaching the question, only about 25% succeeded in achieving the majority of marks.
The algebraic approach based on forming an equation with digits was generally more successful. Candidates who formed such an equation usually proceeded to the full solution, at worst losing a mark for failing to specify clearly that the leading digit of zero (which is possible in a PIN number) cannot satisfy their equation. Some modular arithmetic techniques used to solve this equation without even short casework were impressive.
It was also possible to solve the problem using estimation and/or cryptogram-style column multiplication techniques to eliminate most of the possibilities for individual digits and then carefully considering the remaining cases. The most efficient of such approaches started with reducing possibilities for the leading digit.
Very few candidates taking the casework route succeeded in covering all the cases rigorously enough to build a watertight mathematical proof, hence the low average mark for this approach.

Q4

This geometry question was successfully solved by about one-third of the candidates. Those who made a start of splitting the large triangle in half and noticing similarity usually successfully proceeded to complete the full proof of the statement in the question with only occasional algebraic slips seen.
When candidates used triangle similarity, some were vague on which triangles are similar and many failed to justify it by quoting the appropriate similarity rule. This omission, while not invalidating the script when otherwise clear and correct, cost the candidates several marks.
A small number of candidates took an alternative approach of forming an equation for total triangle area using similarity and area scale factors. This, while more complex than the model solution, could also lead to a correct proof, and some of these scripts have been awarded full marks.
The question asked to prove the given statement for all triangles and all positive integer values of n. Scripts that considered only one or two values of n (e.g. 2 or 3) and those that made assumptions about the triangle (e.g. equilateral or right-angled) received only few marks. It is worth noting, however, that most of these solutions could be easily extended to cover the general case.

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Q5

This challenging question consisted of two separate parts:
1) Showing that $k < 45$, typically by noticing that any two of the three numbers 30, 60 and 90 must belong to different groups;
2) Splitting all the whole numbers 1 to 88 into two groups of 44 in a way described in the question and showing that this split indeed satisfies the requirements.
Many candidates came up with the key mathematical insight necessary to solve part (1), but only few appreciated there is a second part to the problem, which was necessary to achieve more than four marks for this question. A small number of scripts included a weaker bound for k based on a different set of three numbers where no two of them can share a group; these scripts were awarded a lower mark.
Even those candidates who attempted part (2) and achieved higher marks, usually struggled to formally justify correctness of their given split. Hence only six scripts were awarded full marks.
The approach outlined above was essentially the only way that led to a successful solution; scripts that tried different approaches failed to make meaningful progress in solving the problem.

Q6

The final question on the Cayley paper turned out to be more approachable than we had expected with over 100 scripts awarded at least eight marks.
The key to solving the problem was exploring parities of numbers of disks of each colour during the procedure. Many candidates noticed, typically through experimentation, that it is impossible to make all the three numbers even, or all three odd, at the same time. Some scripts stopped at this stage, receiving few marks, but those that formally proved this observation typically proceeded to obtain the full solution.
When proving that an invariant always holds, one needs to prove that it holds at the start of the procedure and that any individual step of the procedure retains it.
A large number of candidates simulated the procedure until they were left with only blue discs. This approach, even when written out in detail, could only yield one mark. This is because it only shows it is possible for the last disc to be blue, not that it is always the case, regardless of the sequence of performed moves.

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