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Hamilton 2024 Markers' Report

By the time candidates are in year 10, many of the better candidates are used to what is expected from Olympiad-style problems, although some will be attempting this for the first time. All candidates deserve to be commended for their resilience and creativity when approaching these problems. Many very fine solutions were submitted. There were some candidates who do not quite grasp the level of explanation that was needed, with several not writing enough. A helpful guide here is, "if you gave this solution to someone else in your class, would they understand what you did at each step?" It means that statements often need a short note of justification. A handful of candidates write long essay-type answers and should be encouraged to keep their solutions to short statements of facts or logical deductions, with reasoning.

The paper proved to be quite well-balanced, although slightly more challenging than previous years, bringing the distribution of marks more in line with Olympiads for older students. Question 1 was very accessible. Question 2 proved more difficult than normal, as geometry often is for students at this age. Questions 3 and 4 were a bit trickier, but many were able to engage with the problems. Questions 5 and 6 required a bit more insight to solve the problems and were testing for the strongest candidates. Candidates should be pleased by achieving one or two correct solutions and be encouraged to enjoy attempting difficult maths problems.

In analysis problems, students must check that they have covered all options to complete a solution. The number of errors of basic arithmetic was surprising: clearly some students are quite reliant on their calculators!

Question 1

Although this question requires nothing more than distance, speed and time, it is an unusual situation in using predicted time rather than actual time of a journey. This is something many will be familiar with in using satellite navigation, but leads to some unexpected situations, like the scenario in the question.

With the unknowns defined in the question, many were able to get started. It was pleasing how many could interpret the situation fully and get to the equations. Only a few made mistakes with the relationship between distance, speed and time. From there many were able to cope with the simultaneous equations, despite them not being linear. At this age, these sorts of equations may be unfamiliar, but concepts like substitution would be familiar and this was handled well.

A few candidates solved the equations, but failed to check what the question asked for and did not quite finish, but, on the whole, this question was dealt with well.

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Question 2

This question proved quite difficult for many and, indeed, scored lower than the question after.
Many defined a useful length with an unknown. Given that the question asked for an angle, a scaled version of the diagram would have the same angle and it was acceptable to define a length with a useful number.
With not much to add to the diagram, it was necessary to add an appropriate line to the diagram. As is often the case when finding an angle between two lines neither of which is horizontal or vertical on the diagram, it may be useful to mark in a horizontal or vertical line to the diagram. The missing angle is then the difference between two angles, both of which are between a horizontal line and another line.
For those who did that, they also needed to either know an appropriate trigonometric ratio or recognise that a triangle was half of an equilateral triangle to make further progress. Many did know a ratio and could then complete the problem.
A few applied the sine rule and were able to complete it by this method, but this was not the expected approach and knowledge of the sine rule was only useful in addition to the original insight into the problem.

Question 3

This proved to be quite a difficult question. While most candidates attempted the question, and many took an algebraic approach, the challenge was correctly forming two simultaneous equations and then manipulating them to draw conclusions.
To start with, it is essential to define what the variables mean. In forming the equations, it is also necessary to explain where they come from instead of just writing them down. Many scripts had marks deducted for missing out this step.
At this point, many candidates struggled with the idea that the simultaneous equations did not have unique solutions and added or subtracted equations and substituted multiple times. Some went round in circles, but a good number eliminated systematically to find relationships between pairs of the unknowns. Those who got this far were often able to draw conclusions about the values the unknowns could take because of the coprime nature of the coefficients of the respective unknowns.

Question 4

This question proved very popular to attempt and seemed to engage many candidates quite well. Given the nature of the problem it was quite accessible to many, with only knowledge of basic index laws required to access the problem.
The key to reaching the higher marks was to address the case where the base number was 3, the other cases being simple to check, with enumeration only required for one case. The result that the last digit of powers of 3 form a periodic sequence seemed new to many candidates, but was generally stated by most, but also justified by many. Some then had problems with the stacking powers and went wrong in the case where the first power was 2, wrongly assuming that the next power also had to be even rather than simply being greater than 1.

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There was still the counting to be completed. With the small number of cases required, many listed the relevant cases and completed the problem. Some missed cases, so thorough checking is required. Those who tried to count by a method sometimes counted the same case as being different and ended up slightly over the correct total.

Question 5

This question did not require a long write-up, but needed some insight into the problem. This is found by asking the right questions while trying examples out.

One of those questions might involve dealing with primes. Students may be familiar with the idea that if the constant term of a monic quadratic is prime then the factorisation is easy to find and one of the brackets contains the number 1. That means that the coefficient of x must be specifically 1 more than the prime. From here it is possible to deduce directly that a pair of consecutive numbers can be dealt with in this way and the whole problem can be solved.

If it hasn't been solved yet, it then might follow to ask whether the coefficient of x can be prime. This is clearly easier, but the constant term ends up being higher unless one of the factors contains the number 1 in the bracket. Again, this leads to constant term being 1 lower than the coefficient of x. Given the restriction on the numbers available, the higher primes, like 97, might cause problems if you do anything else. With that in mind, it seems, if there is a solution, it must use that property.

A good number did find this property. However, not all were able to explicitly link it to a solution to the problem of removing all the numbers from the board. These problems need a precise approach which demonstrates how to do this and those who did not missed out on the highest marks. However, there was a pleasing number of successful attempts.

Question 6

It is unusual to see two geometry questions, but this was a nice challenge for the end of the paper. With some reasonable logic, it was quite accessible to those who were confident with geometry and good reward for those who enjoy it and are willing to try these types of problems. Like most geometry problems, the higher marks were only accessible to those who essentially solved the problem.

The main starting point seemed to be to realise that if one side length is the sum of two other side lengths then it would be useful to divide that side into two parts, each of which is equivalent to one of the other side lengths. If done in the right way it leads to the sensible next step of creating isosceles triangles and this got some credit. Many who did this were able to complete the problem.

At this point knowledge that the angle bisector of the apex of an isosceles triangle is the same as the perpendicular bisector of the opposite side, which meant the three lines are all perpendicular bisectors of the sides of a triangle and coincide at the circumcentre. Some quoted this as a standard result. Given that it was the last question, many preferred to justify this as well. It was pleasing to see as many complete solutions as there were for a last question on geometry.

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