UK Maths Trust

Andrew Jobbings Senior Kangaroo

Wednesday 20 November 2024

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England & Wales: Year 13 or below | *Scotland: S6 or below* | *Northern Ireland: Year 14 or below*

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil** to record your answer to each problem as a three-digit number from 000 to 999.
   Pay close attention to the instructions on the Answer Sheet that shows how to code your answers.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer.
   There is no penalty for giving an incorrect answer.
7. **Your Answer Sheet will be read by a machine.** Do not write or doodle on the sheet except to mark your chosen options. The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, doodle, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way, or reject the answer sheet.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Andrew Jobbings Senior Kangaroo should be sent to:

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1. The seventh day of an unknown month in an unknown year is a Tuesday. The nth day of the following month is the first Tuesday of that month. What is the product of all the possible values of n?

2. Each of the following expressions has a different value. Which value is greatest?

$$2 - 0 \times 2 + 4 + 11 \times 20$$
$$2 + 0 \times 2 \times 4 + 11 \times 20$$
$$2 \times 0 + 2 \times 4 + 11 \times 20$$
$$2 \times (0 + 2 + 4) + 11 \times 20$$

3. Kevin begins with eleven stones and a mallet. Every time he hits a stone with the mallet it breaks into exactly eight pieces. Kevin keeps hitting stones with his mallet, but stops before he has 1000 stones. What is the largest number of stones he could end up with?

4. The number 35 can be written as the sum of four distinct primes in two different ways: $35 = 2 + 3 + 13 + 17$ and $35 = 2 + 3 + 11 + 19$.
What is the *smallest* number that can be written as the sum of four distinct primes in two different ways?

5. What is the value of $|\sqrt{123} - 456| + |\sqrt{123} + 456|$?
The modulus function, $|x|$ is given by $|x| = x$ for $x \geq 0$, and $|x| = -x$ for $x < 0$.

6. The equation $x^2 - 70x = 2024$ has solutions a, b with $b > a$. What is the value of $10b - a^2$?

7. The triangle ABC is such that $AB = 7$, $BC = 3$, and $CA = 5$.
Furthermore, we are given that $\angle ACB = 120°$.
The area of triangle ABC is M. What is the value of M^2, correct to the nearest integer?

8. At a football match between Glasgow Belles and Doncaster City, five supporters each made a prediction:

1. The game will end in a draw;
2. Glasgow will score;
3. Glasgow will win;
4. Glasgow will not lose;
5. Exactly five goals will be scored.

What is the maximum number of these predictions that could simultaneously come true?

9. The number $N = 24^{2024} + 24^{2025}$ has some factors between 1 and 25.
What is the sum of all those integers between 1 and 25 which are *not* factors of N?

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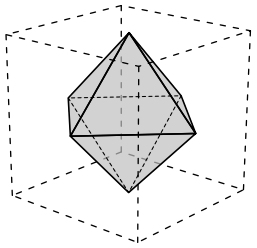
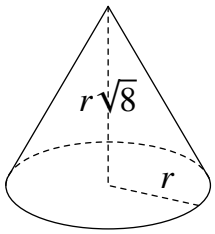
10. An octahedron is inscribed in a cube of side-length 12. The vertices of the octahedron are at the centre of the faces of the cube.
The surface area of the octahedron is $a \times \sqrt{b}$, where b is a prime number. What is the value of $a + 10b$?



11. The number $L = \frac{24!}{20!}$ is one less than a square, S^2.
What is the value of S?

12. A solid right circular cone whose base has radius r metres has total surface area (including the circular base) of A metres squared and a volume of V metres cubed.
The perpendicular height of the cone is $r\sqrt{8}$ metres.
Given that $\frac{V}{A} = \sqrt{5000}$, what is the value of r?



13. Wil goes on a run. Her pedometer is unfortunately rather basic, and records a distance of 40 km, to the nearest integer, and an average speed of 10 km/h, to the nearest integer.
Wil works out her greatest possible and least possible times, and expresses the difference of these times as a fraction in its lowest terms, $\frac{a}{b}$.
What is the value of $a + 2b$?

14. The function f has the property that $f(x + y) = f(x) \times f(y)$ for all values of x and y.
Given that $f(x) > 0$ for all values of x, and also that $f(24) = 4096$, what is $f(18)$?

15. Four friends named Ada, Brin, Chris and Debbie are each carrying a different amount of money. These amounts are £111, £222, £333 and £444.

Ada says: I am neither carrying the most nor the least.

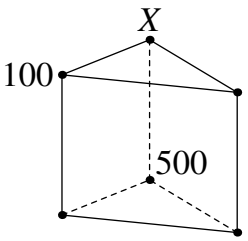
Brin says: I am not carrying the least.

Chris says: I am carrying the most.

Debbie says: I am carrying the least.

Exactly one of the friends is lying. How many letters does the lying friend have in her/his name?

16. The prism in the picture is formed of two triangles and three squares. The six vertices are assigned labels 100, 200, 300, 400, 500, and 600 in such a way that the sum of the four labels of the vertices of each square is the same for all three squares. Labels 100 and 500 are shown.
What number is at the vertex labelled X?



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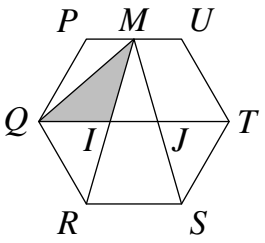
17. The parabola $y = x^2 - 10x + 24$ has two distinct x-intercepts and a y-intercept.
The circle which goes through these three intercepts meets the parabola at a fourth point, P.
What is the distance of P from the origin?

18. Zarah has a bag containing exactly four blue counters and exactly four red counters, and a large supply, outside the bag, of extra blue and red counters.
She may, on any given *move*, do one of four different things:

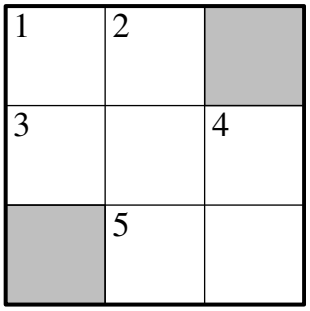
- Remove a blue counter from the bag;
- Remove a red counter from the bag;
- Add a blue counter to the bag;
- Add a red counter to the bag.

At each move, she selects one of those four options at random and with equal probability.
The probability that after four consecutive moves, Zarah's bag again contains exactly four blue counters and exactly four red counters is $\frac{V}{W}$ when written in its lowest terms.
What is the value of $10V + W$?

19. A regular hexagon $PQRSTU$ is shown. The midpoint of UP is M. Point I is the intersection between line segments RM and QT, and point J is the intersection between line segments SM and QT. The area of triangle RMS is 920. What is the area of triangle QMI?



20. Each cell in this cross-number can be filled with a non-zero digit so that all of the conditions in the clues are satisfied.
The digits used are not necessarily distinct.



ACROSS
1. A prime.
3. The answer to this Kangaroo question.
5. Shares a two-digit common factor with 1 DOWN.

DOWN
1. The sum of two different sixth powers.
2. 101 less than a square number.
4. The sum of all the digits in this crossnumber, once correctly completed.

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