UK Maths Trust

Cayley 2025 Markers' Report

Many candidates in year 9 are not that used to what is expected from Olympiad-style problems. However, there were many well-written and creative solutions and it was impressive how many could cope with solving problems which are not of the type they see in school and require different skills. Some candidates don't quite understand the level of explanation that is needed. Many write waffling solutions, which explain trivial things in unnecessary detail, but others write too little and miss key points. The students should ask themselves whether the solution could be understood by someone who has never seen the correct solution and is following theirs and there would be no gaps in their thinking.

The paper proved to be quite accessible at all levels, at a level of challenge comparable with the previous year. This may be the first time for many of the candidates that they have not scored high marks. Students should be heartened by achieving one or two correct solutions and encouraged to enjoy the challenge of difficult but interesting problems. Questions 1 and 2 were very approachable for all, although not always easy to get high marks. Geometry can put candidates off and they often do not know what is required to show a given result. Question 3 was accessible, but getting the equations was not easy. Question 4 was a problem which for many will be the first time they've encountered this type of maths. Question 5 is an accessible idea, but challenging at this age to get the ideas rigorously into a complete solution. Question 6 was more unusual but some managed to complete it.

In questions where the idea is to find what values are possible, showing all the possible ones and demonstrating why there are no others is a problem for many candidates.
For optimisation problems, students must check that they have shown that nothing better can be achieved as well as demonstrating that the optimum can be achieved.
There were some errors of basic arithmetic and students need to be confident working with numbers without relying on their calculators!

Question 1

This proved to be an accessible opener to the paper with 19% of candidates achieving full marks and a further 45% of candidates achieving six marks or more. Among those scoring fewer marks, many had found the correct final answer but lacked explanation. For any Olympiad problem, easy or hard, it is vital that candidates make their method clear to markers and they do not leave out any necessary details.
It was pleasing to see that the vast majority of candidates could translate the problem into conventional algebra and form a correct equation that reflected the information given. From here, there were two main approaches seen by markers, one was to focus on a and the other was to focus

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on b. There were several ways to reduce the number of cases under consideration, using divisibility arguments, but often the most successful candidates were able to simply check all ten cases for the value of a or b and obtain the final answer in a way that made it clear it was unique.
Of the candidates achieving a high mark but not full marks, there was one very common omission in their solutions which was that $b = 0$ was not considered. It is important to remember that 0 is valid digit choice and is also a multiple of all whole numbers. Where $a = 0$ was not considered explicitly, we were more lenient as it was the first digit of a two-digit number.
This question could have had an even higher success rate. If a candidate reduces the problem to a small set of cases in need of checking, it is important that each case is dealt with fully and candidates do not skip over necessary working, even if it feels repetitive.

Question 2

Many candidates were able to make good progress on this question by making use of isosceles triangles to deduce correct angle facts and combine geometrical facts to come up with the correct value of x. Some students got stuck at this stage, and it was common to see an asserted value for angle *ACB* without geometrical foundation. Those who substituted the value of x into their expressions for angle values usually correctly realised that triangle *ADC* was equilateral and were able to finish the proof. Geometrical reasoning accompanying the steps in student working was usually sufficient, though some candidates lost marks when writing equations without accompanying annotations, for example when using the sum of interior angles in a triangle.
A significant number of mathematically confident candidates appeared not to read the question carefully and were able to complete the proof without ever finding the value of angle *BDC* also asked for in the question, meaning only partial marks could be awarded.

Question 3

The majority of students attempted this question with considerable success, though many lacking any real justification. Unfortunately, when an incorrect value for x was found and used to calculate a they could still get the correct final answer.
Not on the mark scheme was another valid method not involving the value of x, they could find v in terms of a and x and a in terms of x then dividing v by a giving the final height. This was also given full credit where it was seen.

Question 4

This question naturally divides itself into two parts: firstly, showing that it is not possible to fit more than 1600 tiles; secondly, finding a way to fit 1600 tiles into the space.
Many candidates managed to show that no more than 1600 tiles will fit by considering areas, but were unable to find a way to fit 1600. Most of these candidates thought that they would have to "stick the tiles back together" and tile the grid with squares of side length $\sqrt{3}$. This method only results in 1568 tiles fitting in the grid, and although there were some attempts to squeeze more tiles in after these 1568, it is ultimately not possible to get the optimum number of tiles with this method.

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Of the candidates who were able to find the construction which gets 1600 tiles, most of them were relatively successful in getting a solution to the problem. However, there were some common pitfalls that resulted in many of these candidates missing out on full marks.

Some candidates were not fully clear in their description of the construction, for example failing to explain why we can definitely fit 400 copies of the square of side length $\sqrt{6}$ into the large square.

When explaining why it is impossible to do better than 1600 tiles, some candidates assumed that the first 1600 tiles would be placed in the way that they had arranged them and argued that the remaining space would not fit any more tiles in it. This is not a valid chain of reasoning, as they failed to show that a completely different arrangement would not be better.

Question 5

The PIN combinations piqued many students' interests, with most candidates making some progress that showed they understood the set up and specialised with particular cases. Many found some (or even all) solutions; however, at Olympiad level this in itself was insufficient to gain more than 1 mark. Some students misinterpreted the question (for example believing the PIN to have 4 digits), so we once again urge students to read the question carefully.

We were looking for a logical argument which guaranteed a process for finding all PINs. These fell into two broad categories. The first was algebraic and involved writing the numbers as a sum of the hundreds, tens and ones digits. With simplification, this produced a product equalling to 1110 which restricts possible cases to combinations of factors. Many students who took this approach scored full marks. Omissions were not excluding numbers with a '0' digit (impossible by the nature of the question) or not explicitly stating why some factor pairs of 1110 were not considered. Often students correctly ignored factor pairs which gave a two-digit k (impossible by the nature of the question) but did not mention they were doing so. It is important students understand this is necessary as failure to explicitly exclude such k means a solution is open to the criticism that not all cases have been considered, which in other questions potentially leads to a very low mark.

A more common approach involved case work from the start. Students either considered every possible k or every possible starting digit. This was very laborious as it involved checking many subcases and explaining their conclusions for each one. Not many candidates who chose this method scored high marks as there were often cases not considered or rejected without a clear explanation.

Question 6

This was a difficult problem and few candidates were able to come up with a truly general strategy for ordering the penguins correctly. Often, successful solutions involved organising the emperors on the left first and then showing how the kings and gentoos can also be put in the correct place (while temporarily disrupting the emperors). There were also a few different correct strategies suggested and all were given full credit as long as they worked in general. This was a question many students could have a go at even if they were unable to come up with a general strategy and some candidates even included nice drawings of penguins! Many students who thought they had solved the problem were focusing on particular arrangements of penguins and relying on specific penguin orderings already being present as part of their strategy. In terms of writing-up style, it is worth noting that a written solution in the context of a Maths Olympiad need not be written in prose: in this case, it was a

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lot easier to follow solutions which made use of symbolic ways of denoting what shuffles were being suggested (e.g. by abbreviating penguins of different types as E, K and G) than solutions which relied entirely on words to describe their strategy.

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