UK Maths Trust

Hamilton 2025 Markers' Report

By the time candidates are in year 10, many of the better candidates are used to what is expected from Olympiad-style problems, although some will be attempting this for the first time. All candidates deserve to be commended for their resilience and creativity when approaching these problems. Many very fine solutions were submitted. There were some candidates who do not quite grasp the level of explanation that was needed, with several not writing enough. A helpful guide here is, "if you gave this solution to someone else in your class, would they understand what you did at each step?" It means that statements often need a short note of justification. A handful of candidates write long essay-type answers and should be encouraged to keep their solutions to short statements of facts or logical deductions, with reasoning.

The paper proved to be quite tough in comparison with recent years. It was well-balanced, although question 1 was more challenging than normal, which brought the marks down. The rest of the paper was more in line with other Olympiad papers. Candidates found question 2 more difficult to write up both parts completely. Questions 3 was well attempted on the whole. Question 4 was accessible and many were able to engage with the problem. Question 5 was more challenging again, but there were good solutions. Question 6 was testing for the strongest candidates and no complete solutions were given. It may be that candidates were able to solve the problem, but ran out of time given the difficulty of the earlier questions compared to previous papers. Candidates should be pleased by achieving one or two correct solutions and be encouraged to enjoy attempting difficult maths problems.

In analysis problems, students must check that they have covered all options to complete a solution. For optimisation problems, students must check that they have shown that nothing better can be achieved as well as demonstrating that the optimum can be achieved.

The number of errors of basic arithmetic was surprising: clearly some students are quite reliant on their calculators!

Question 1

This was a tough question 1, but there were many good attempts.
The most successful candidates started by finding expressions for the volume of water as this was invariant throughout the problem. They then eliminated variables until finding a ratio in x and y, which they could then substitute back into previous expressions to find a solution.
A large number of candidates found a solution of 18. This generally came about by attempting to work proportionally with height and volume. This did not work out as this does not take into account the portion of the cylinders that remain above the surface of the water.

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Some candidates ended up with equations which do not work dimensionally. It is worth noting that if you multiply an area by a length you expect to end up with a volume, but if multiplying two areas together you are then working in a 4D hyperspace which would not help when solving the problem. When forming an equation candidates should take note that all of the terms have the same dimensions, so not subtracting a length from a volume for instance. This is part of a larger part of maths/physics called dimensional analysis which can tell a lot about a formula from the units of each term.
A lot of candidates with otherwise good solutions lost a mark due to not justifying where their initial equations came from. Having a diagram or some reference to using the cross-sectional area was generally sufficient.

Question 2

This was found much more difficult than was intended. There are two essential parts to the problem. The contestant must show that it is possible to tile the floor with 300 rectangles and 16 squares (construction). Then they must show that it is impossible to tile the floor with fewer than 316 tiles (lower bound). To score highly, both parts must be nearly complete.
The construction was done successfully by about half the contestants. It is natural to place 20 rectangles in a column of width 80 and length 9. Repeating 15 such columns leaves a column of width 80 and length 5 which can be filled with 16 squares. But there are plenty of other arrangements of 300 rectangles and 16 squares that also work.
The lower bound is more difficult. Most of the candidates were making unwarranted assumptions – for example, that we had to have the rectangles and squares in neat columns and/or rows, rather than any possible pattern.
The only successful approach was to consider the total area, leading to the equation $36r + 25s = 140\times80$, where r is the number of rectangles and s is the number of squares. Observing that r is divisible by 25 quickly leads to $r = 300$ and $s = 16$ as the best solution.
This argument could be expressed less formally. Once we have a tiling that works, we can only improve it by replacing 36 squares by 25 rectangles, possibly rearranging the other tiles to accommodate the new rectangles. But we have to be careful with such arguments. It is not true that we have to replace a particular arrangement of squares (often 4 by 9) by a particular arrangement of rectangles (often 5 by 5).
Several candidates managed to prove the lower bound but did not attempt the construction. But the latter is essential to the argument. For example, if the problem involved 1 by 36 rectangles rather than 4 by 9 rectangles, our equation still shows that it is impossible to tile the floor with fewer than 316 tiles. But it is also impossible to tile it with 300 rectangles and 16 squares.

Question 3

Many candidates did this question well by using the expected method of similar triangles and the associated scale factor. Some students were reckless in not matching the vertices correctly when stating the similarity, and sometimes were led astray. Some students discussed scale factors without reference to the similarity of triangles. One or two students
attempted to use Cartesian coordinates, but the angle conditions are difficult in that context.

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There were good attempts using area scale factors as well. Pleasingly, few tried to consider the parallelogram to be a rectangle or to assign specific values to missing angles. Use of mathematical notation for similarity was quite sketchy. It would help candidates in preparation for these papers to know standard notation for similarity and congruence.
Note that any triangle AXC with $AX{:}\,AC = 5{:}\,7$ can be embedded uniquely in this configuration. The angle *XAB* (equal to angle *DAC*) is not determined by the data in the question. Those students who attempted to calculate the angles in the figure were chasing rainbows.

Question 4

This proved to be a reasonably popular question. However, only around half of those who attempted it were able to score any marks. The key was to take an algebraic approach, representing the PIN in terms of its digits and deriving an equation which can then be solved by considering factors. Unfortunately, a large number of candidates did not think of this and instead tried to analyse different cases according to the value of k and the first and/or last digit of the PIN. It is possible to carry out a very careful analysis and reduce the problem to checking fewer than 50 different numbers; however, only a handful of candidates were able to get anywhere near this, and almost everyone who took this approach scored at most one mark for finding one PIN that works.
Of those who took the more fruitful approach, a large majority were able to see it through to the end. Some stumbled because they didn't realise that they could treat $(1000a + 100b + 10c + d)$ as a single number and manipulate equations with it, but instead resorted to looking at individual digits. An interesting misconception was observed when candidates tried to do this: many thought that the first digit of the new PIN would be $ka + 2$, ignoring the fact that there could be "carries" when adding four-digit numbers.
Candidates who found the correct solution often lost marks at the end for claiming that 1817 is prime, which it is not. This turns out not to matter, because 9085 has no other four-digit factors, but this needed to be justified rather than just stated.

Question 5

Many candidates attempted this question, but only about a quarter of them reached the threshold for 10-.
The high-scoring scripts largely worked along the lines of the two methods given in the official solutions. That is, the candidate either used casework starting with the observation that 5 cannot divide both $n - 3$ and $n + 3$, or they listed all numbers of the form $\pm 2^a 5^b$ within the appropriate range and noted those pairs that differed by six. Where explanation was lacking, especially in the latter case, marks were docked.
Some scripts used properties of the units digit to narrow down the search (for example, when one of $n \pm 3$ is divisible by 10, the other is a power of two ending in 4 or 6). Mistakes included claims that if $n + 3$ ends with a 5 then $n - 3$ ends with a 9 (what about $n = 2$?), and that if $n - 3$ is a power of two ending in 4 then $n + 3$ works (we saw several 67s and 1027s).

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All successful attempts used the factorisation $n^2 - 9 = (n - 3)(n + 3)$. Without it, the candidate was left checking for perfect squares of the form $n^2 = 2^a 5^b + 9$, which was typically attempted only for $n \leq 45$, forgetting that the upper bound of 2025 was on n and not on $n^2 - 9$.

As is traditional, many candidates failed to deal effectively with small cases, which in this problem was when $n - 3 < 0$.

A large number of candidates did not realise, or could not express clearly, that $\frac{1}{k}$ is a terminating decimal if and only if $k = \pm 2^a 5^b$. Some thought that k needed only to be a multiple of 2 and 5; others thought that once factorised into $(n - 3)(n + 3)$ the 2s must all go into one factor and the 5s into the other. This alone proved to be a useful way of separating the candidates, as befits a question at the harder end of the paper.

Question 6

There were not many serious attempts at this question, possibly because of running out of time. The candidates who did have a proper try found this question very hard, primarily because they didn't manage to see the best approach to the problem. On this sort of question, it's a good idea to draw diagrams, starting with small cases. A number of candidates worked out the correct answer for $n = 3$ and $n = 5$, but then didn't manage to find the optimal sequence that works for all odd values of n.

The key part of the question was to find and prove a strategy to give a lower bound for M, as well as finding a sequence of moves which leads to the bound being achieved. Candidates need to be careful about what happens in the corner since the corner cells aren't accessible to the mole.

Several candidates misunderstood the question, assuming that Mo cannot go back through cleared cells; these candidates went straight from the centre cell to an edge and then stopped, obtaining a simpler, but, unfortunately, incorrect, formula. Mo only stops when no more cells can be dug up; the mole goes back through the cleared cells to dig up any other cells according to the given rule.

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