UK Maths Trust

Maclaurin 2025 Markers' Report

Both candidates and their teachers will find it helpful to know something of the general principles involved in marking Olympiad-type papers. The preliminary paragraphs therefore serve as an exposition of the 'philosophy' which has guided both the setting and marking of all such papers at all age levels, both nationally and internationally.

What we are looking for, essentially, is solutions to problems. This approach is therefore rather different from what happens in public examinations such as GCSE and A level, where credit is given for the ability to carry out individual techniques regardless of how these techniques fit into a protracted argument. Such marking is cumulative; a candidate may gain 60% of the available marks without necessarily having a clue about how to solve the final problem. Indeed, the questions are generally structured in such a way as to facilitate this approach, divided into many parts and not requiring an overall strategy for tackling a multi-stage argument.

In distinction to this, Olympiad-style problems are marked by looking at each question synoptically and deciding whether the candidate has some sort of overall strategy or not. An answer which is essentially a solution, but might contain either errors of calculation, flaws in logic, omission of cases or technical faults, will be marked on a '10-minus' basis. One question we often ask is: if we were to have the benefit of a two-minute interview with this candidate, could they correct the error or fill the gap? On the other hand, an answer which shows no sign of being a genuine solution is marked on a '0-plus' basis; up to 3 or 4 marks might be awarded for particular cases or insights. It is therefore important that candidates taking these papers realise the importance of the rubric about trying to finish whole questions rather than attempting lots of disconnected parts.

The paper was similar in difficulty to recent papers. Question 1 proved more difficult than other papers, but there were more successful attempts at questions 3 and 4 than would be expected from questions at this level, so the mark structure ended up similar to previous papers.

Question 1

Most students answered this question following the method of the model solution, by finding expressions for the volumes of water with the two orientations of the box and solving the resulting quadratic in x. A few used an alternative approach which begins by calculating the volume of the part of the box protruding above the water when its non-square face is on the bottom.
Among students who essentially solved the problem, many lost a mark for not clearly rejecting the negative solution of the quadratic. Some failed to calculate the quantity asked for in the question, either stopping with the depth of water after the box is removed, or with the drop from the first scenario instead of the further drop from the second scenario.

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A key to the usual solution of the problem was realising that the depth of water in the first scenario in the question is 10 and in the second scenario is 10-1=9. Some students missed these facts, introducing an extra variable for the depth of water in one of the scenarios. Some students also introduced a variable for the volume or height of the cylindrical tank. Usually, the extra variables resulted in equations not having a unique solution, but a few students succeeded in eliminating the extra variables to get back to an equation in x alone.

A few students did not realise that part of the box protrudes above the water surface in the second scenario in the question. Drawing a diagram would be helpful! This, or other errors, sometimes led to absurd answers such as two contradictory equations, no drop in water level, or a drop larger than the initial depth of water. It is a good idea to check whether the final answer is plausible – again a diagram would help with this.

A few students incorrectly assumed that x and certain other lengths must be integers and tried to use divisibility properties, usually involving factors of 36. By simple bounds for x, they were usually able to deduce the correct value of x i.e. 12, but this is not a valid solution because the question did not guarantee that any quantities are integers.

Question 2

This question required students to realise that an optimal strategy was to subtract the units' digit each time, if this is possible. Students could then proceed to explain the simple example where Alice starts with 125 and then explain how the strategy generalises to any three-digit number. A key point that students needed to emphasise was that the values would alternate between multiples of 10 and those which aren't and provide a good explanation why. There were two main stumbling blocks in this question: the first is that students misread the question and assumed any number between 1 - 9 inclusive could be subtracted, rather than it needing to be one of the digits in the three-digit number; the second was not explaining that three-digit numbers that are multiples of 10 are the only possibility.

There were many good responses to this question and a lot of students were able to organise a clear and succinct argument to explain the reasoning around their solution. The best responses ensured that all cases were covered and that there was a solid argument for each step that needed to be justified. The key components of such a proof were: that an optimal strategy is to subtract the units digit; the value written will always alternate between multiples and non-multiples of ten; and how different starting values enforce different strategies. Some were unclear that whatever move is made from a multiple of 10, it is possible to return to a multiple of 10 next and just gave specific examples. Some stated that the strategy was to return to a multiple of 10 by following a move subtracting k with a move subtracting $10 - k$, but did not check that this was always a possible move. The reason it works is that subtracting k from a multiple of 10 leaves a units digit of $10 - k$, which can then be subtracted in the next move to return to a multiple of 10. This is an important detail which was omitted by many.

Question 3

This question had several aspects to it and attempts at all parts were required to reach the higher marks.

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Firstly, candidates had to deduce the last two digits of powers of 7. Few realised they could use just the last two digits from the previous power to calculate the next one and even fewer realised that once the last two digits were 01 the pairs of digits would be repeated pairs because the zeroth power of 7 already ends 01. Those who used only the last digit were not able to access many marks.
Secondly, candidates needed to realise there were only three possible sums of four pairs to lead to an ending of 00. Some considered two pairs adding to 50, which was fine, but many of those had problems counting later. Many did not take into account that {01,49} and {07,43} is the same as {07,43} and {01,49} and double-counted those. Some treated {07,43} and {01,49} the same as {01,49} and {01,49}, so missed that the some arrangements of the ones with repeats are being double-counted.
Thirdly, candidates needed to consider the possible arrangements of the pairs. Several missed this possibility. Mostly, candidates who did consider this were aware of factorials for this and dealing with repeats where they existed. However, those who dealt with pairs summing to 50 often kept the first pair as the first two values in the sum, forgetting that the pair could be in any of the four positions in the sum. Even those who did remember this often had problems when there were two pairs of the same numbers. They generally found the options for the solutions within each pair, then found the number of permutations, forgetting that with repeats two identical solutions can be found from this process, so there is overcounting.
Fourthly, candidates needed to consider the actual solutions for the unknowns rather than just the pairs. Some missed details like 0 being a possibility as a solution, but mostly this was fine.
An alternative approach separated cases involving repeated digits. This was much harder to do and rarely led to a correct solution and the mistakes were not easy to spot. In many cases, the number of solutions where all four digits were different was found correctly, so the solution scored at least 7. The main problem came in the case with a value appearing twice alongside two different values. Whether that value was one of the set of three with the same remainder modulo 4 (like 0, 4 and 8) or a set of two (like 2 and 6) affected the number of options for the other values, but this was often not accounted for correctly.

Question 4

Many candidates spotted the similar triangles *ABX* and *ADM* and were able to use the isosceles triangle *AXM* to find the ration *AX : AM* by quoting the ratios of side lengths in a triangle with angles 30°, 60° and 90°, or using the sine rule in a triangle with angles 30°, 120° and 30°.
After this many candidates struggled to correctly relate the ratio of *BX : AD* in triangles *ABX* and *ADM* and could not progress further.
In general, those who drew a clear diagram of similar triangles and established the scale factor were more successful. This got them to the 10- mark scheme, but there were penalties for lack of reasoning. In particular, not justifying why angles *ABX* and *ADM* are equal and inaccuracy in the statement of similarity not paying attention to the order of the vertices in the triangles were penalised. A small number of candidates stated the ratio *BX : BC* instead.

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Question 5

This was an interesting problem to mark as there were quite a few approaches that worked, but it was also possible to reach the correct numerical answer through a sequence of unjustified claims or by assuming that maximising m (or n) necessarily resulted in maximising $m + n$. There were also approaches that didn't lead anywhere, such as looking at the parity of m and n; some candidates tried these and necessarily scored very poorly, but it seemed far more common for candidates to try a potentially fruitful approach but fail to lack the rigour required at this stage in the contest. The overall impression was that most of the candidates who attempted a solution had understood what was going on to some extent, but relatively few understood the need to be rigorous in their reasoning to ensure that they had definitely found the right answer. This feeling was reinforced by the number who claimed the answer was 110 because they had decided that it was sufficient to maximise n.

Question 6

This question proved quite engaging for many candidates despite it being question 6. Quite a few were able to find a way of creating a 5 by 5 square with four triangles and a 1 by 1 square and most of those completed the 13 by 13 square. Some tried to describe it and were unclear, so a diagram would have been quicker and clearer.

Proving impossibility for the second part was completed in a variety of ways. The most common approach was to prove that a 1 by 1 square could not be at an edge, so that left a 5 by 5 square at the centre. However, proof that it left a 5 by 5 square was missing in many cases and just stated. The possibility that the centre might be an irregular shape was not considered. This was penalised heavily.

The key colouring approach was seen only once and an alternative colouring approach was attempted as well, which was pleasing to see.

In both parts, candidates tried arguments along the lines of stating that the area matches so it must be possible. Just because a reason which might be a problem in other cases is not a problem here does not lead to a conclusion that it must be possible.

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