UK Maths Trust

Andrew Jobbings Senior Kangaroo

© 2025 UK Mathematics Trust

a member of the Association Kangourou sans Frontières

proudly sponsored by [XTX] MARKETS

Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

Enquiries about the Andrew Jobbings Senior Kangaroo should be sent to:

challenges@ukmt.org.uk

supported by Overleaf

1. The product $2 \times 3 \times 5 \times 7 \times 11 \times 13$ has remainder r when divided by 100.

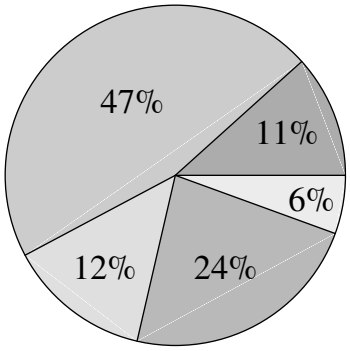
What is the value of $200 - r$?

SOLUTION **170**

The product is $30 \times 1001 = 30030$, so $r = 30$ and $200 - r = 170$.

2. The pie chart shows how exactly 2000 children in a particular town get to school. Note that the percentages in this pie chart are exact, without any rounding. Approximately twice as many use a bicycle as use the bus. Roughly the same number come by car as walk. The rest come by train.

How many children use the train?



SOLUTION **120**

The only two sectors of approximately the same size are those marked 12% and 11%, so these must represent 'car' and 'walk', in some order. Of the remaining sectors, those marked 47% and 24% are the only ones in an approximate ratio of 2 : 1, so these must be 'bicycle' and 'bus' respectively. Therefore the sector marked 6% represents the children who use the train. Therefore, the train is used by 6%, which is 120 children.

3. In a sum of five three-digit numbers 'ABC'+'BCD'+'CDE'+'DEA'+'EAB' = 2664, the letters A, B, C, D and E represent digits, which are not necessarily distinct.

What is the value of $10(A + B + C + D + E)$?

SOLUTION **240**

Writing 'ABC' as $100A + 10B + C$, and similarly for the other numbers, the sum of the five numbers is $111(A + B + C + D + E)$, which is equal to 2664.
It follows that $A + B + C + D + E = \frac{2664}{111} = 24$, so that $10(A + B + C + D + E) = 240$.

© 2025 UK Mathematics Trust

4. Kitty claims that for every positive integer n, the product $(5n+1)(6n-1)(n^3-2)$ is not divisible by 7.

What is the smallest three-digit value of n which is a counterexample to Kitty's claim?

SOLUTION **102**

We consider the remainder of each of the three constituent brackets $(5n+1)$, $(6n-1)$, and (n^3-2) when each is divided by 7.
100 has remainder 2 when divided by 7, so the brackets will have remainders of $5\times 2+1=11$ (remainder 4), $6\times 2-1=11$ (remainder 4), and $2^3-2=6$ respectively. As none of these are zero, the product will not be divisible by 7.
Similarly, 101 has remainder 3 when divided by 7, so the brackets will have remainders of $5\times 3+1=16$ (remainder 2), $6\times 3-1=17$ (remainder 3), and $3^3-2=25$ (remainder 4) respectively. As none of these are zero, the product will not be divisible by 7.
However, 102 has remainder 4 when divided by 7. The first bracket then has remainder $5\times 4+1=21$, so is divisible by 7.

5. The integers a, b, and c are such that $1\le a\le b\le c$ and $abc=2^{16}$.

What is the largest possible value of b?

SOLUTION **256**

The case $a=1$ has $b=c=2^8$ as a possible solution, so the largest possible value of b is at least $2^8=256$.
Were $b>2^8$, then $c>2^8$ also, and $abc>2^{8+8}=2^{16}$.
So 256 is the largest possible value of b.

6. Three fair dice are rolled. The first dice has three red faces, two white faces and one blue face. The second dice has two red faces, two white faces and two blue faces. The third dice has one red face, two white faces and three blue faces. The probability that, following rolling, all three dice will show the same colour is $\frac{K}{1080}$.

What is the value of K?

SOLUTION **100**

The probability of obtaining three reds is $\frac{3}{6}\times\frac{2}{6}\times\frac{1}{6}=\frac{6}{216}$.
The probability of obtaining three whites is $\frac{2}{6}\times\frac{2}{6}\times\frac{2}{6}=\frac{8}{216}$.
The probability of obtaining three blues is $\frac{1}{6}\times\frac{2}{6}\times\frac{3}{6}=\frac{6}{216}$.
So, the probability of obtaining three of the same colour is $\frac{6}{216}+\frac{8}{216}+\frac{6}{216}=\frac{20}{216}=\frac{100}{1080}$.

© 2025 UK Mathematics Trust

7. What is the value of $\dfrac{(2025+20)^2+(2025+25)^2-(2025-20)^2-(2025-25)^2}{1000}$, rounded up to the next integer?

SOLUTION **365**

The numerator may be written as

$$\begin{aligned}2045^2-2005^2+2050^2-2000^2 &= (2045-2005)(2045+2005)+(2050-2000)(2050+2000)\\ &= 40\times 4050+50\times 4050\\ &= 90\times 4050 = 9\times 40500.\end{aligned}$$

The expression is therefore equal to $\frac{9\times 40500}{1000} = 9\times 40.5 = 364.5$.
Rounded up to the next integer, this is 365.

8. The first four digits of a 128-digit number are 2025.

How many digits does the square of this number have?

SOLUTION **255**

We recall that an m-digit number may be written in standard form as $a\times 10^{m-1}$, where $1\le a<10$.
In standard form, the 128-digit number, N, is such that $2.025\times 10^{127}\le N<2.026\times 10^{127}$.
Therefore, $2.025^2\times 10^{254}\le N^2<2.026^2\times 10^{254}$.
Since $1<2.025^2<2.026^2<10$, this is in standard form and will have length $254+1=255$ digits.

9. Nineteen integers, which are not necessarily distinct, are arranged in a circle such that the sum of the integers in any nine consecutive positions always has the same value, 2025.

What is the value of the largest of the nineteen integers?

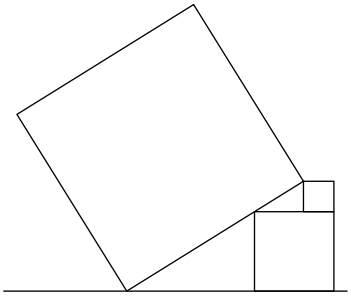
SOLUTION **225**

Label the integers in turn c_1, c_2, c_3, etc., with c_{19} adjacent to c_1.
Hence $c_1+c_2+c_3+c_4+c_5+c_6+c_7+c_8+c_9 = c_2+c_3+c_4+c_5+c_6+c_7+c_8+c_9+c_{10}$, from which we conclude that $c_1=c_{10}$.
A similar argument can be used to show $c_2=c_{11}$, $c_3=c_{12}$, $c_4=c_{13}$, $c_5=c_{14}$, $c_6=c_{15}$, $c_7=c_{16}$, $c_8=c_{17}$, $c_9=c_{18}$, $c_{10}=c_{19}$, $c_{11}=c_1$, $c_{12}=c_2$, $c_{13}=c_3$, $c_{14}=c_4$, $c_{15}=c_5$, $c_{16}=c_6$, $c_{17}=c_7$, $c_{18}=c_8$, and $c_{19}=c_9$.
We can conclude that all nineteen integers are equal to each other.
The largest integer is (and indeed all of the integers are) $\frac{2025}{9}=225$.

© 2025 UK Mathematics Trust

10. Three squares are shown in the picture. The area of the smallest square is 9, and the area of the middle-sized square is 81.

What is the area of the largest square?



SOLUTION **720**

The smaller right-angled triangle has hypotenuse $\sqrt{3^2 + 6^2} = \sqrt{45}$.
The larger right-angled triangle (with height 9), is similar to the smaller right-angled triangle (with height 3), so has hypotenuse $\frac{9}{3} \times \sqrt{45} = 3 \times \sqrt{45}$.
Hence the side of the largest square is $\sqrt{45} + 3 \times \sqrt{45} = 4 \times \sqrt{45}$.
Therefore the largest square has area $4^2 \times 45 = 16 \times 45 = 720$.

11. The sequence k_n is defined by $k_1 = 19$, $k_2 = 11$ and $k_{n+2} = k_n + k_{n+1}$ for $n \geq 1$.

How many of the first 2025 terms in the sequence are even?

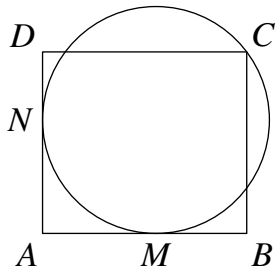
SOLUTION **675**

When we evaluate the terms of the sequence in order, we obtain $k_3 = 19 + 11 = 30$, $k_4 = 11 + 30 = 41$, $k_5 = 30 + 41 = 71$, $k_6 = 41 + 71 = 112$, etc.
We see that the pattern 'odd, odd, even, odd, odd, even' will recur indefinitely. Since 2025 is a multiple of 3, exactly one-third of the first 2025 terms will be even.
The number of terms in the sequence that are even is $\frac{2025}{3} = 675$.

© 2025 UK Mathematics Trust

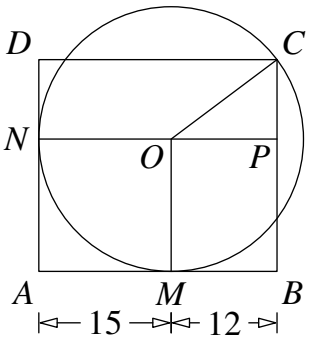
12. A rectangle $ABCD$ and a circle are shown in the diagram. Sides AB and AD of the rectangle are tangent to the circle at points M and N respectively.
The circle passes through C.
The length of AM is 15 and the length of MB is 12.

What is the area of the rectangle?



SOLUTION **648**

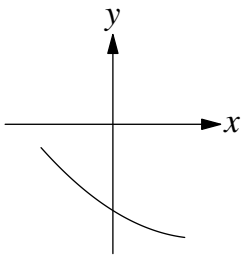
Let O be the centre of the circle and consider the radii OC, OM, and ON as shown, with NO extended to meet BC at P.
Since $OP = MB = 12$ and $OC = ON = MA = 15$, the right-angled triangle COP has side lengths that form a 9–12–15 Pythagorean triple.
Now, $AB = AM + MB = 15 + 12 = 27$, and $BC = BP + PC = MO + PC = 15 + 9 = 24$.
Therefore the area of the rectangle is $27 \times 24 = 648$.



13. Part of the parabola with equation $y = ax^2 + bx + c$ is shown in the diagram.

How many of the following expressions have positive values?

a	b	c	ab	bc	ca	$b+c$	abc

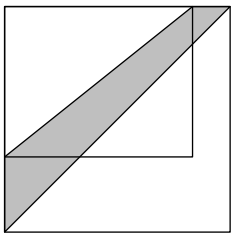


SOLUTION **003**

As the parabola is "U"-shaped we know that $a > 0$. The y-intercept is negative, so $c < 0$. Completing the square, we obtain $y = a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right) + c$. Since the parabola's minimum occurs for a positive value of x, and this must correspond with $\left(x + \frac{b}{2a}\right) = 0$, we know that $\frac{b}{2a} < 0$. As $a > 0$, we can conclude $b < 0$.
It follows that $ab < 0$, $ac < 0$, $bc > 0$, $b + c < 0$, and $abc > 0$.
Therefore, there are three positive quantities: a, bc, and abc.

© 2025 UK Mathematics Trust

14. The length of one side of a rectangular park was increased by 20%, while the length of the other side was increased by 50%. After the changes the park is now square in shape. The shaded area shown between the two diagonals in the diagram is 210.



What was the original area of the park before the changes were made?

SOLUTION **525**

Let the original rectangle have longer side of length x, and shorter side of length y.
The area of the original rectangle is xy and the area of the square is $1.2x \times 1.5y = 1.8xy$.
The shaded area is $0.5 \times 1.8xy - 0.5 \times xy = 210$.
Therefore, $0.4 \times xy = 210$, and hence the original area of the park was $xy = \frac{210}{0.4} = \frac{2100}{4} = 525$.

15. The simultaneous equations (1) and (2) have at least two distinct (x, y) solutions.

$$2x + 5y = a - b \qquad (1)$$
$$4x + 10y = a + b \qquad (2)$$

The simultaneous equations (3) and (4) also have at least two distinct (x, y) solutions.

$$3x - 2y = a - b + 4 \qquad (3)$$
$$9x - 6y = a + 4b - 1 \qquad (4)$$

What is the value of $10a + b$?

SOLUTION **403**

Equations (1) and (2) can have more than one distinct solutions only if one equation is a multiple of the other. Therefore, $a + b = 2(a - b)$, from which we can conclude that $3b = a$.
Similarly, equations (3) and (4) can have more than one distinct solutions only if one equation is a multiple of the other. Therefore, $a + 4b - 1 = 3(a - b + 4)$.
Solving $3b = a$ and $a + 4b - 1 = 3(a - b + 4)$ simultaneously yields $a = 39$ and $b = 13$. The value of $10a + b$ is $10 \times 39 + 13 = 403$.

© 2025 UK Mathematics Trust

16. The six interior angles of a convex non-regular hexagon, measured in degrees, are a, $a+d$, $a+2d$, $a+3d$, $a+4d$, and $a+5d$. All six angles are integer multiples of $10°$.

What is the value of $7a+11d$?

SOLUTION **710**

When you consider the interior angle sum of the hexagon, you obtain $6a+15d=720$. Since both a and $a+d$ are multiples of 10, it follows that d is also a multiple of 10.
We know that d cannot be 0 as the hexagon is not regular.
If $d=10$ then $6a+150=720$ and $a=95$, which isn't a multiple of 10.
If $d=20$ then $6a+300=720$ and $a=70$.
If $d=30$ then $6a+450=720$ and $a=45$, which isn't a multiple of 10.
If $d\geq 40$ then $a+5d\geq 200$, making the hexagon non-convex.
Therefore $a=70$, $d=20$, and hence $7a+11d=7\times 70+11\times 20=490+220=710$.

17. The navigation app on my smartphone gives distances in both miles and kilometres. When planning a journey, I notice the distance in miles, m, and the distance in kilometres, k, are such that $m=k-438$.

How long does my planned journey in miles work out to be if you use the approximation 1 mile = 1.6 kilometres?

SOLUTION **730**

Substituting $k=1.6m$ into the equation gives $m=1.6m-438$.
This has solution $m=730$, which is the length of the planned journey.

18. A teacher works out the number of different ways, W, she can choose two students from Class A. She also works out the number of different ways, V, she can choose two students from Class B. Class A has four more students in it than class B. She notices that $W=3V$.

How many students are there in class A?

SOLUTION **010**

Let the number of students in class A be P.
The number of ways, W, of choosing two students from class A is $\frac{P\times(P-1)}{2}$.
The number of ways, V, of choosing two students from class B is $\frac{(P-4)\times(P-5)}{2}$.
Since $W=3V$, then $\frac{P\times(P-1)}{2}=3\times\frac{(P-4)\times(P-5)}{2}$.
Simplifying this gives the equation $P^2-13P+30=0$. This has solutions $P=10$ or $P=3$.
Only in the $P=10$ case does class B have a positive number of students. Hence there are ten students in class A.

19. An *almost number* is defined as an integer which is divisible by exactly eight elements of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

What is the largest *almost number* below 1000?

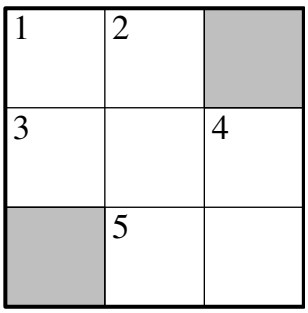
SOLUTION **840**

By considering the prime factorisation of those composite elements of the set ($4 = 2^2$, $6 = 2 \times 3$, $8 = 2^3$, and $9 = 3^2$), we can find the lowest common multiple of the set of all nine numbers. This is $2^3 \times 3^2 \times 5 \times 7 = 2520$.

Almost numbers are numbers (or multiples of numbers) obtained on dividing 2520 by 2, 3, 5 or 7. These are, respectively, 1260, 840, 504 and 360, or their multiples. The only multiple of any of these which is less than 1000 is 720.

The largest three-digit almost number is 840.

20. Each cell in this cross-number can be filled with a non-zero digit so that all of the conditions in the clues are satisfied. The digits used are not necessarily distinct.



What is the sum of the answers to the clues for

1 ACROSS, 3 ACROSS and 5 ACROSS?

ACROSS

1. A factor of 1221.
3. A multiple of 17.
5. A prime.

DOWN

1. A factor of 1221 different from 1 ACROSS.
2. A cube.
4. A prime.

SOLUTION **457**

$1221 = 3 \times 11 \times 37$, so its only two-digit factors are 11, 33, and 37. Since the solutions to 1 ACROSS and 1 DOWN are distinct and share a tens digit, they must be 33 and 37 in some order.

Consider the solution to 1 ACROSS to be 33. Then 1 DOWN is 37 and 2 DOWN is 343, since this is the only three-digit cube starting with a 3. The only multiple of 17 that then fits the clue for 3 ACROSS is 748. 4 DOWN must be either 83 – with 5 ACROSS being 33 (which is composite and therefore does not satisfy the clue), or 89 – with 5 ACROSS being 39 (which is also composite and therefore does not satisfy the clue). So 1 ACROSS cannot be 33.

So the solution to 1 ACROSS must be 37. Then 1 DOWN is 33 and 2 DOWN is 729, since this is the only three-digit cube starting with a 7. . The only multiple of 17 that then fits the clue for 3 ACROSS is 323. 4 DOWN must either be 31 - with 5 ACROSS being 91 (which is composite and therefore does not satisfy the clue), or 37 - with 5 ACROSS being 97 which is indeed prime. This completes the crossnumber.

The sum of the answers to the clues for 1 ACROSS, 3 ACROSS and 5 ACROSS is $37+323+97 = 457$.

© 2025 UK Mathematics Trust