UK Maths Trust

Senior Mathematical Challenge

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Solutions

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
E	C	B	D	A	D	D	B	B	C	C	B	E	C	A	B	C	A	A	E	E	D	129	140	528

1. What is the product of two numbers whose sum is 20 and whose difference is 26?

A 6 B 46 C 12 D -46 E -69

SOLUTION **E**

Let the two numbers be x and y with $x > y$. Therefore $x + y = 20$ and $x - y = 26$. Adding these equations gives $2x = 46$, so $x = 23$ and $y = -3$. The product is $23 \times -3 = -69$.

2. Alana buys 4 apples and 5 pears for £3.20. At the same prices, 5 apples and 4 pears cost £3.10.

What is the total cost of one apple and one pear?

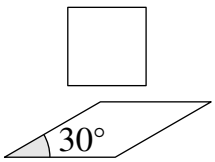
A 50p B 60p C 70p D 80p E 90p

SOLUTION **C**

Let a pence be the price of an apple and p pence be the price of a pear. Therefore $4a + 5p = 320$ and $5a + 4p = 310$. Adding these equations gives $9a + 9p = 630$, therefore the total cost of one apple and one pear is 70 pence.

3. The square and the rhombus with interior angle of 30° have the same area.

What is the ratio of the side-length of the square to the side-length of the rhombus?



A 1 : 1 B $1 : \sqrt{2}$ C $\sqrt{3} : 2$ D 2 : 3 E 1 : 2

SOLUTION **B**

Let the side-length of the square be x and the side-length of the rhombus be y. As the areas are equal, we have $x^2 = y \times y \times \sin 30°$. Therefore $x^2 = y^2 \times \frac{1}{2}$ which rearranges to $\sqrt{2}x = y$. The required ratio is then $1 : \sqrt{2}$.

4. $2^x = \sqrt{4\sqrt{4\sqrt{4}}}$. What is the value of x?

A 3 B $\frac{5}{2}$ C 2 D $\frac{7}{4}$ E $\frac{3}{2}$

SOLUTION **D**

Since $2^x = \sqrt{4\sqrt{4\sqrt{4}}}$, squaring both sides gives $2^{2x} = 4\sqrt{4 \times 2}$. Expressing both sides as powers of 2, $2^{2x} = 2^2 \times 2^{\frac{3}{2}}$. Equating the powers, $2x = 2 + \frac{3}{2}$ and therefore $x = \frac{7}{4}$.

5. A sequence of numbers has the property that for every positive integer n, the mean of the first n terms of the sequence is $\frac{1}{n}$.

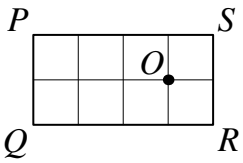
What is the 1000th term of this sequence?

A 0 B $\frac{1}{999}$ C $\frac{1}{1000}$ D $\frac{1}{1001}$ E $\frac{1}{1000000}$

SOLUTION **A**

In order to create a sequence for which the mean of the first n terms is $\frac{1}{n}$, the sum of the first n terms must be 1. The sequence must therefore be 1, 0, 0, 0, 0, ... , with first term 1 and all subsequent terms, including the 1000th term, having value 0.

6. Rectangle $PQRS$ is made from eight 1 by 1 squares as shown in the diagram. At point O, four of those squares meet. The rectangle is rotated through 180° about point O. Point P travels k times as far as point R during that rotation.

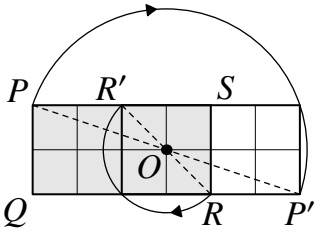


What is the value of k?

A $\sqrt{2}$ B $\sqrt{3}$ C 2 D $\sqrt{5}$ E $\sqrt{6}$

SOLUTION **D**

Both P and R travel the length of a semicircle during the rotation of 180°. Using Pythagoras' Theorem, the length of OR is $\sqrt{2}$, so R moves $\pi\sqrt{2}$ to reach R'. Similarly, the length of OP is $\sqrt{10}$, so P moves $\pi\sqrt{10}$ to reach P'. Therefore $k = \frac{\pi\sqrt{10}}{\pi\sqrt{2}} = \sqrt{5}$.



7. A bag contains 50 yellow and 25 purple beads which are to be arranged into five groups of 15.

What is the largest possible number of the groups having more purple than yellow beads?

A 0 B 1 C 2 D 3 E 4

SOLUTION **D**

In order to have more purple beads than yellow in a group of 15 beads, we require there to be 8 or more purples. As there are 25 purples available and $25 = 3 \times 8 + 1$, it is possible to create just 3 groups with more purples than yellows.

8. The number N is the largest 10-digit integer, all of whose digits are different, which is divisible by 7.

What is the remainder when N is divided by 10?

A 0 B 1 C 2 D 3 E 4

SOLUTION **B**

The largest number that can be created with ten different digits is 9876543210, which by division can be shown to be 2 more than a multiple of 7. Swapping any two adjacent digits in the large number reduces its value by a multiple of 9. Swapping the tens and ones digits reduces the number by exactly 9, which in combination with the original remainder of 2 now produces a number $N = 9876543201$ which is a multiple of 7. The remainder when N is divided by ten is then 1.

9. A string of letters is created by applying the following rule in each iteration: Any 'a' is replaced by 'ab', and any 'b' is replaced by letter 'a'. Alison writes down letter 'a' in line 1, then she applies the rule and writes down 'ab' in line 2, and 'aba' in line 3.

How many 'a's will appear in line 10 if Alison keeps applying the same rule correctly?

A 34 B 55 C 64 D 89 E 128

SOLUTION **B**

The number of 'a's on line k is the sum of the number of 'a's on line $k-1$ and the number of 'b's on line $k-1$. The latter is equivalent to the number of 'a's on line $k-2$. As line 1 is a single 'a', the numbers of 'a's at each stage are the Fibonacci numbers. We therefore require the 10th Fibonacci number which is 55.

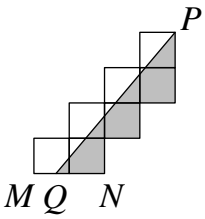
10. How many seven-digit numbers are there such that each digit is either 4, 5 or 6 and each pair of adjacent digits differ by 1?

A 12 B 18 C 24 D 30 E 32

SOLUTION **C**

Building a seven-digit number from the left, first notice that both a 4 and a 6 must be followed by a 5 (one choice). However a 5 can be followed by either a 4 or a 6 (two choices). Therefore there are $1 \times 2 \times 1 \times 2 \times 1 \times 2 \times 1 = 8$ possible seven-digit numbers which start with a 5. Similarly there are $2 \times 1 \times 2 \times 1 \times 2 \times 1 \times 2 = 16$ possible seven-digit numbers which start with a 4 or a 6. In total, there are $8 + 16 = 24$ numbers which fit the constraints given.

11. Seven squares of unit length are aligned edge-to-edge as shown. A straight line is drawn from P to Q, where Q is the point on the line MN such that the shaded area formed is half the total area of the seven squares.

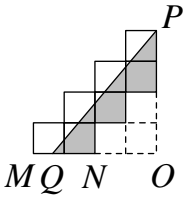


What is the ratio $MQ : QN$?

A 1 : 2 B 2 : 3 C 3 : 5 D 5 : 8 E 8 : 13

SOLUTION **C**

In order to find the position of Q, we can consider the area of triangle QOP which is made from half of the original 7 squares plus the 3 extra, shown with dotted lines in the diagram. Therefore $\frac{1}{2}OQ \times OP = \frac{13}{2}$. Since $OP = 4$, $OQ = \frac{13}{4}$. Then $MQ = 4 - \frac{13}{4} = \frac{3}{4}$ and $NQ = \frac{5}{4}$. The ratio $MQ : QN = 3 : 5$



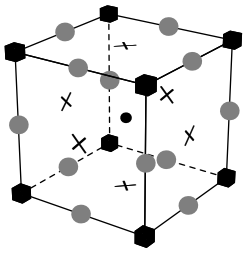
12. A cube has a dot at each vertex, at the midpoint of each edge and at the midpoint of each face.

How many straight lines can pass through exactly 3 dots?

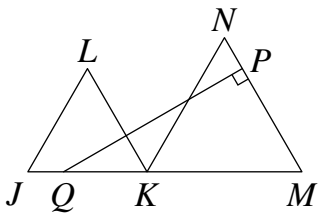
A 24 B 36 C 48 D 60 E 72

SOLUTION **B**

Dots can either be at a vertex (type V), on the midpoint of an edge (type E) or at the midpoint of a face (type F). The diagram shows these using different symbols in order to distinguish between them. Since there is no dot in the centre of the cube, any line of three dots must lie entirely on one face of the cube. The lines must therefore be of the form VEV, VFV or EFE. Type VEV lie along each edge of which there are 12, type VFV lie on each diagonal of the six faces (two per face) and type EFE lie on each of the six faces (again, two per face). In total there are $12 + 2 \times 6 + 2 \times 6 = 36$ possible lines.



13. JKL and KMN are equilateral triangles with side-lengths 20 and 26 respectively. Points J, K and M lie on a straight line. Points P on NM, and Q on JK, are such that JQ and NP have the same length and QPM is a right angle. The length of PQ is $t\sqrt{3}$.



What is the value of t?

A 24 B 23 C 22 D 21 E 20

SOLUTION **E**

Since $\angle QMP = 60°$, being inside equilateral triangle KMN, and $\angle QPM = 90°$ (given), the side-lengths of triangle QPM are in the ratio $1 : \sqrt{3} : 2$. As the length PQ is given as $t\sqrt{3}$, length $PM = t$ and length $QM = 2t$. We are also given that PN and QJ are the same length; let this be x. As MN has length 26, $t + x = 26$. As JK has length 20, JM has length 46 and so $2t + x = 46$. The difference between these equations gives $t = 20$.

14. Consider the possible expressions formed when each □ is filled with a + or a − sign in $\square\frac{1}{1}\square\frac{1}{2}\square\frac{1}{3}\square\frac{1}{4}\square\frac{1}{5}\square\frac{1}{6}$. For example, $-\frac{1}{1}+\frac{1}{2}-\frac{1}{3}-\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$ which has value $-\frac{43}{60}$ is one of the 2^6 possible expressions.

Among these expressions, what is the smallest possible positive value?

A $\frac{1}{60}$ B $\frac{1}{30}$ C $\frac{1}{20}$ D $\frac{1}{10}$ E $\frac{1}{5}$

SOLUTION **C**

Expressing each fraction using a common denominator of 60 gives $\frac{60}{60}, \frac{30}{60}, \frac{20}{60}, \frac{15}{60}, \frac{12}{60}, \frac{10}{60}$. Adding them all produces the largest total of $\frac{147}{60}$. In order to produce the smallest possible positive value we need to partition the fractions into two sets whose totals are as close as possible to half of this. The value sought is the difference between these totals. By inspection of the six numerators, this is achieved by $\frac{75-72}{60} = \frac{3}{60} = \frac{1}{20}$.

15. The mean of n numbers is n, $n > 2$. One of the numbers is $4n$ and another is $n - 6$. The other numbers are all equal to k.

Which of these expressions gives k in terms of n?

A $n - 3$ B $n - \frac{5}{2}$ C $n - 2$ D $n - \frac{3}{2}$ E $n - 1$

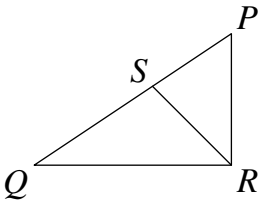
SOLUTION **A**

Since the mean of n numbers is n, the total of those n numbers is n^2. Two of the numbers have a total of $4n + (n - 6) = 5n - 6$, therefore the remaining $(n - 2)$ numbers, each of whose value is k, have a total of $n^2 - 5n + 6$. Hence $k(n - 2) = n^2 - 5n + 6$ and factorising the RHS as $(n - 3)(n - 2)$ gives $k = n - 3$.

16. Triangle PQR has a right angle at R. The lengths of QR and RP are $3\sqrt{5}$ and $2\sqrt{5}$ respectively and $\angle QRS = 45°$.

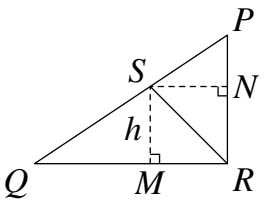
What is the area of triangle PSR?

A 5 B 6 C $3\sqrt{5}$ D 7.5 E $7\sqrt{2}$

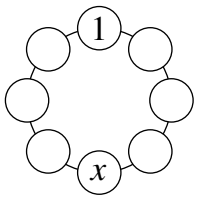


SOLUTION **B**

Drawing perpendiculars from S to QR and PR produces a quadrilateral $SMRN$. Since $\angle QRS = 45°$, $SMRN$ is a square. Let the length of each side of the square be h. The area of triangle PQR is equal to the sum of the areas of triangles SQR and PSR. Hence $\frac{1}{2} \times 3\sqrt{5} \times 2\sqrt{5} = \frac{1}{2}3\sqrt{5} \times h + \frac{1}{2}2\sqrt{5} \times h$ which simplifies to $15 = \frac{5\sqrt{5}}{2}h$ and $h = \frac{6}{\sqrt{5}}$. Therefore the area of triangle $PSR = \frac{1}{2}2\sqrt{5} \times \frac{6}{\sqrt{5}} = 6$.



17. The numbers 2, 3, 4, 5, 6, 7 and 8 are to be placed in the empty circles so that (i) there is one number in each circle, and (ii) the sum of the numbers in each pair of adjacent circles is prime.

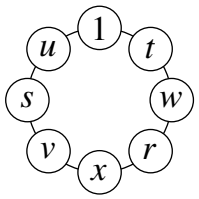


How many possible values are there for x?

A 0 B 1 C 2 D 3 E 4

SOLUTION **C**

Let the values in the eight circles be 1, r, s, t, u, v, w, and x as shown. In order that the numbers in adjacent pairs of circles have a sum which is prime, r, u, v and t must be even and s, w and x must be odd. Consider first the position of the number 8. Since 8 may not be next to 1 (9 is not prime), either $r = 8$ or $v = 8$. Without loss of generality, choose $r = 8$. Consider next the position of the number 7. Since 7 may not be next to 8 (15 is not prime), $s = 7$. Consider next the position of the number 2. Since 2 may not be next to 7 (9 is not prime), $t = 2$.



So $u = 4$ or 6. If $u = 4$ then $v = 6$, therefore $x = 5$ (to avoid $6 + 3 = 9$) and $w = 3$. If however $u = 6$ then $v = 4$, therefore $x = 3$ (to avoid $4 + 5 = 9$) and $w = 5$. There are therefore two possible values of x which allow the diagram to be completed.

18. Let $g(x) = x^{\frac{1}{n}}$. Each of x, a and b are positive real numbers and n is a positive integer.

Which of the following is a simplification of $g(a) \times g(b^2) \times g(a^3) \times g(b^4) \times \ldots \times g(a^{2n-1}) \times g(b^{2n})$?

A $a^n b^{n+1}$ B $a^n b^{n+2}$ C $a^{n+1} b^{n+2}$ D $a^{2n} b^{2n+2}$ E $a^{2n+1} b^{2n+2}$

SOLUTION **A**

First note that the sum of the first n odd numbers $1+3+\ldots+(2n-1)$ is the square number n^2. The sum of the first n even numbers is n more than that, which is $n^2+n = n(n+1)$. Now considering function g, $g(a)\times g(b^2)\times g(a^3)\times g(b^4)\times\ldots\times g(a^{2n-1})\times g(b^{2n}) = a^{\frac{1}{n}} b^{\frac{2}{n}} a^{\frac{3}{n}} b^{\frac{4}{n}} \times \ldots \times a^{\frac{2n-1}{n}} b^{\frac{2n}{n}}$ $= a^{(\frac{1}{n}+\frac{3}{n}+\ldots+\frac{2n-1}{n})} \times b^{(\frac{2}{n}+\frac{4}{n}+\ldots+\frac{2n}{n})}$. Using the earlier results this is equal to $a^{\frac{n^2}{n}} \times b^{\frac{n(n+1)}{n}} = a^n b^{n+1}$.

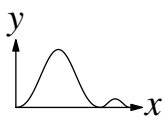
19. The largest integer less than or equal to x is represented by $\lfloor x \rfloor$. For example $\lfloor 5.8 \rfloor = 5$ and $\lfloor 3 \rfloor = 3$.
For how many positive integers x does $\lfloor x^{\frac{1}{2}} \rfloor + \lfloor x^{\frac{1}{3}} \rfloor = 8$?

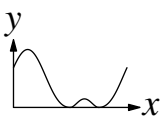
A 9 B 10 C 11 D 12 E 13

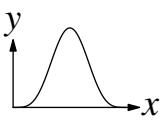
SOLUTION **A**

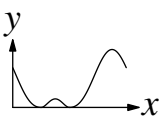
Considering the sequence of positive integers x, the first value of x for which $\lfloor x^{\frac{1}{2}} \rfloor = k$ for some integer k, is a square. The last value of x for which $\lfloor x^{\frac{1}{2}} \rfloor = k$ is one less than the next square. For example, $\lfloor x^{\frac{1}{2}} \rfloor = 6$ when $36 \leq x < 49$. Similarly, $\lfloor x^{\frac{1}{3}} \rfloor = 2$ when $8 \leq x < 27$. This is a case where $6 + 2 = 8$ but there is no overlap between the two sets of values of x. The only sets which have values of x in common and for which $\lfloor x^{\frac{1}{2}} \rfloor + \lfloor x^{\frac{1}{3}} \rfloor = 8$ are $\lfloor x^{\frac{1}{2}} \rfloor = 5$, when $25 \leq x < 36$ and $\lfloor x^{\frac{1}{3}} \rfloor = 3$, when $27 \leq x < 81$. The nine common values of x are 27, 28, 29, 30, 31, 32, 33, 34 and 35.

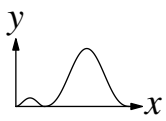
20. Which of the following is a sketch of the graph of $y = 1 + \sin x \cos x - \sin x - \cos x$ for $0° \leq x° \leq 360°$?

A 

B 

C 

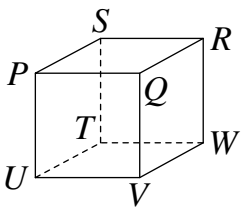
D 

E 

SOLUTION **E**

By factorising $y = 1 + \sin x \cos x - \sin x - \cos x$ as $y = (1 - \cos x)(1 - \sin x)$ we observe that $y = 0$ when $1 - \cos x = 0$, so $\cos x = 1$ which is so when $x = 0$ and when $x = 360$. Also, we have another point where $y = 0$ when $1 - \sin x = 0$, so $\sin x = 1$ which is so when $x = 90$. The graph in option E is the only one with all of these properties.

21. The diagram shows a clear plastic cube, $PQRSTUVW$. An ant starts at vertex S and walks on the surface of the solid cube. It travels along straight line segments (first to Q, U or W), alternately across the diagonal of a face and then along an edge. An example of a possible five-segment journey is $SQRTSW$.



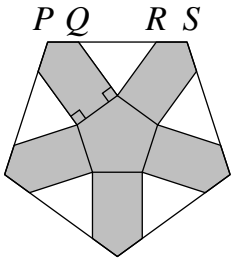
At the end of a journey of 81 segments, which of the following vertices is it impossible for the ant to reach?

A S B Q C U D W E V

SOLUTION **E**

The eight vertices of the cube can be split into two sets, (1) containing Q, S, U and W and (2) containing P, R, T and V. When the ant travels along the diagonal of a face, it moves to a vertex in the same set. When the ant travels along an edge, it moves to a vertex in the opposite set. As a result of the alternating nature of the ant's journey, the sets which contain the vertices at which the ant could be produce a sequence of period 4; (1), (2), (2), (1), (1), (2), (2), (1), ... Since 81 is 1 more than a multiple of 4, the ant can only be at a vertex within (1), and so it can not be at vertex V.

22. As shown in the diagram, a playground is built in the shape of a regular pentagon. In the centre is a smaller regular pentagon, with lines perpendicular to each side at each vertex of the smaller pentagon. The order of the rotational symmetry of the diagram is 5. The side-length of the larger pentagon is 10 m, and $PQ = RS = 2$ m.

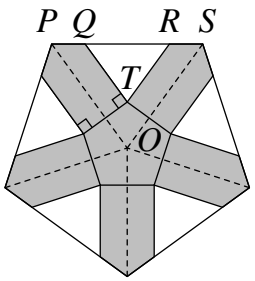


What fraction of the area of the playground is the area of the shaded region?

A $\frac{3}{5}$ B $\frac{5}{8}$ C $\frac{9}{25}$ D $\frac{16}{25}$ E $\frac{81}{125}$

SOLUTION **D**

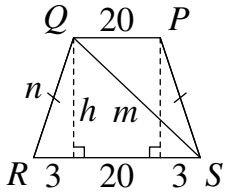
The diagram can be split into five congruent triangles along the dotted lines as shown. At point T, the angle inside the smaller regular pentagon is 108°. Also, $\angle QTR = 360° - (2 \times 90° + 108°) = 72°$. Therefore $\angle QTR = \angle POS$ and hence triangle QTR and triangle POS are similar as their bases lie along the same edge. Since $PS = 10$ and $QR = 10 - 2 \times 2 = 6$, the lengths QR and PS are in the ratio $3 : 5$. The areas of triangles QTR and POS are therefore in the ratio $3^2 : 5^2$. The fraction of the area of the playground which is shaded is then $1 - \frac{9}{25} = \frac{16}{25}$.



23. Let $PQRS$ be a trapezium with QP parallel to RS and with $QP = 20$, $RS = 26$ and $PS = QR = n$, for some positive integer n. The length of the diagonal QS is also a positive integer.

What is the maximum possible value of n?

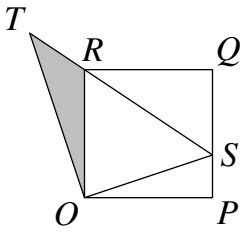
SOLUTION **129**



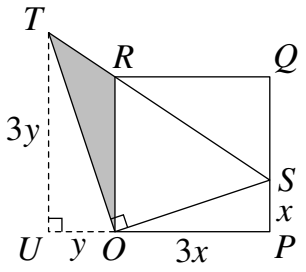
Consider the two right-angled triangles shown, with height h and vertex Q. Applying Pythagoras' Theorem, first to the triangle with hypotenuse QR gives $3^2 + h^2 = n^2$ (1) and then to the triangle with hypotenuse QS gives $23^2 + h^2 = m^2$ (2). Subtracting, (2) – (1) gives $23^2 - 3^2 = m^2 - n^2$. Therefore $520 = (m + n)(m - n)$. As both $m + n$ and $m - n$ are integers, we can consider the factor pairs of 520. Taking first $m + n = 520$ and $m - n = 1$ produces non-integer values of m and n. Moving to $m + n = 260$ (3) and $m - n = 2$ (4) however leads to $2n = 258$ and therefore $n = 129$. Any other factor pairs will have values closer together than 260 and 2 and so will produce a smaller value of n.

24. $OPQR$ is a square. The point S lies on PQ such that $PS : SQ = 1 : 2$. Points S, R and T lie on a straight line and angle TOS is a right angle. The area of the triangle ORT is 30.

What is the area of $OPQR$?

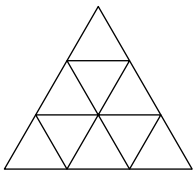


SOLUTION **140**



Since $PS : SQ = 1 : 2$, let $PS = x$ and therefore $OP = 3x$. The length of $OS = \sqrt{10}x$ and the gradient of OS is $\frac{x}{3x} = \frac{1}{3}$. Since $\angle TOS = 90°$, the gradient of OT is -3. Extending PO until point U, directly below T, allows us to show that if $OU = y$ then $UT = 3y$. Therefore the length of $OT = \sqrt{10}y$. We can now express the shaded area in terms of x and y in two ways. Firstly, viewed as a triangle with base OR and perpendicular height OU, its area is $\frac{1}{2} \times 3x \times y$. Secondly, viewed as the triangle TOS with the area of ROS (base OR and perpendicular height OP) removed, its area is $\frac{1}{2} \times \sqrt{10}x \times \sqrt{10}y$ - $\frac{1}{2} \times 3x \times 3x$. Equating these two expressions gives $\frac{3xy}{2} = 5xy - \frac{9x^2}{2}$ and therefore $9x^2 = 7xy$. We are given that the shaded area is 30, so $\frac{1}{2} \times 3x \times y = 30$ and $xy = 20$. The area of $OPQR$ which is $9x^2$ is therefore $7 \times 20 = 140$.

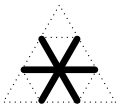
25. This diagram, built from equilateral triangles of side-length 1, has a total of 18 edges of length 1. Each edge is to be coloured red, white or blue. Note that different colourings of the edges of the diagram which can be made the same by rotation or reflection of the diagram are to be considered as different colourings.



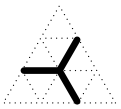
How many different coloured diagrams can be made, given that each equilateral triangle of side-length 1 must have edges of all three colours?

SOLUTION **528**

The problem reduces to the number of ways of colouring the central six lines. Once their colours are chosen, the colours of the extra six lines which close the six central triangles, shown dotted, are uniquely determined. Then there are two ways to colour each of the corner triangles, so $2^3 = 8$ ways once the central six triangles are coloured.



When considering the colouring of the central six lines, one successful strategy is to split the counting into three cases:



(a) The three spokes shown are differently coloured. There are $3! = 6$ ways to do this. In each case, the rest of the colouring of the central hexagon happens uniquely.

(b) The three spokes are all the same colour. There are 3 ways this can happen. The other 3 spokes can then be coloured in 2^3 ways and then the rest of the colouring of the central hexagon happens uniquely.

(c) Two spokes are one colour, and one spoke is another colour. There are 3 ways to choose the positions of the identically coloured spokes, and then 3×2 ways to choose the colours used. Now consider the other three spokes. The colours of two of them are forced, and the third could be one of two possible colours. The rest of the colouring of the central hexagon is then uniquely determined.

This gives $8 \times (6 + (3 \times 2^3) + (3 \times 3 \times 2 \times 2)) = 8 \times 66 = 528$ different ways.