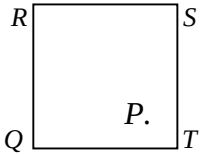
19. Hamish and his friend Ben live in villages which are 51 miles apart. During the summer holidays, they agreed to cycle towards each other along the same main road. Starting at noon, Hamish cycled at x mph. Starting at 2 pm, Ben cycled at y mph. They met at 4 pm. If they had both started at noon, they would have met at 2.50 pm. What is the value of y?

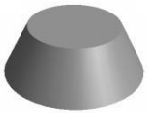
A 7.5 B 8 C 10.5 D 12 E 12.75

20. A point P is chosen at random inside a square $QRST$. What is the probability that $\angle RPQ$ is acute?



A $\frac{3}{4}$ B $\sqrt{2}-1$ C $\frac{1}{2}$ D $\frac{\pi}{4}$ E $1-\frac{\pi}{8}$

21. A frustum is the solid obtained by slicing a right-circular cone perpendicular to its axis and removing the small cone above the slice. This leaves a shape with two circular faces and a curved surface. The original cone has base radius 6 cm and height 8 cm, and the curved surface area of the frustum is equal to the area of the two circles. What is the height of the frustum?

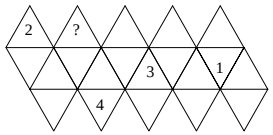


A 3 cm B 4 cm C 5 cm D 6 cm E 7 cm

22. M and N are the midpoints of sides GH and FG, respectively, of parallelogram $EFGH$. The area of triangle ENM is 12 cm^2. What is the area of the parallelogram $EFGH$?

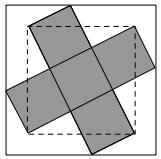
A 20 cm^2 B 24 cm^2 C 32 cm^2 D 48 cm^2 E more information is required

23. The net shown is folded into an icosahedron and the remaining faces are numbered such that at each vertex the numbers 1 to 5 all appear. What number must go on the face with a question mark?



A 1 B 2 C 3 D 4 E 5

24. A figure in the shape of a cross is made from five 1×1 squares, as shown. The cross is inscribed in a large square whose sides are parallel to the dashed square, formed by four of the vertices of the cross. What is the area of the large outer square?



A 9 B $\frac{49}{5}$ C 10 D $\frac{81}{8}$ E $\frac{32}{3}$

25. Four positive integers a, b, c and d are such that

$$abcd + abc + bcd + cda + dab + ab + bc + cd + da + ac + bd + a + b + c + d = 2009.$$

What is the value of $a + b + c + d$?

A 73 B 75 C 77 D 79 E 81

UK SENIOR MATHEMATICAL CHALLENGE

Thursday 5 November 2009

Organised by the **United Kingdom Mathematics Trust**

and supported by

The Actuarial Profession
making financial sense of the future

RULES AND GUIDELINES (to be read before starting)

1. Do not open the question paper until the invigilator tells you to do so.
2. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
3. Time allowed: **90 minutes**.
   No answers or personal details may be entered on the Answer Sheet after the 90 minutes are over.
4. The use of rough paper is allowed.
   Calculators, measuring instruments and squared paper are forbidden.
5. Candidates must be full-time students at secondary school or FE college, and must be in Year 13 or below (England & Wales); S6 or below (Scotland); Year 14 or below (Northern Ireland).
6. There are twenty-five questions. Each question is followed by five options marked A, B, C, D, E. Only one of these is correct. Enter the letter A-E corresponding to the correct answer in the corresponding box on the Answer Sheet.
7. **Scoring rules**: all candidates start out with 25 marks;
   0 marks are awarded for each question left unanswered;
   4 marks are awarded for each correct answer;
   1 mark is deducted for each incorrect answer.
8. **Guessing**: Remember that there is a penalty for wrong answers. Note also that later questions are deliberately intended to be harder than earlier questions. You are thus advised to concentrate first on solving as many as possible of the first 15-20 questions. Only then should you try later questions.

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http://www.ukmt.org.uk

1. What is 20% of 30%?

   A 6% B 10% C 15% D 50% E 60%

2. Which of the following is not a multiple of 15?

   A 135 B 315 C 555 D 785 E 915

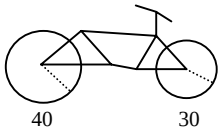
3. What is the value of $1^6 - 2^5 + 3^4 - 4^3 + 5^2 - 6^1$?

   A 1 B 2 C 3 D 4 E 5

4. Steve travelled 150 miles on a motorbike and used 10 litres of petrol. Given that 1 gallon $\approx$ 4.5 litres, roughly how many miles per gallon did Steve achieve on his journey?

   A 10 B 20 C 40 D 50 E 70

5. Boris Biker entered the Tour de Transylvania with an unusual bicycle whose back wheel is larger than the front. The radius of the back wheel is 40 cm, and the radius of the front wheel is 30 cm. On the first stage of the race the smaller wheel made 120000 revolutions. How many revolutions did the larger wheel make?



   A 90000 B 90000π C 160000 D $\dfrac{160000}{\pi}$ E 120000

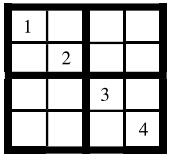
6. A bag contains hundreds of glass marbles, each one coloured either red, orange, green or blue. There are more than 2 marbles of each colour.

   Marbles are drawn randomly from the bag, one at a time, and not replaced.

   How many marbles must be drawn from the bag in order to ensure at least three marbles of the same colour are drawn?

   A 4 B 7 C 9 D 12 E 13

7. A mini-sudoku is a 4 by 4 grid, where each row, column and 2 by 2 outlined block contains the digits 1, 2, 3 and 4 once and once only. How many different ways are there of completing the mini-sudoku shown?

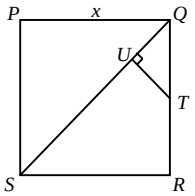


   A 1 B 2 C 4 D 8 E 12

8. The entries to the Senior Mathematical Challenge grew from 87400 in 2007 to 92690 in 2008. Approximately what percentage increase does this represent?

   A 4% B 5% C 6% D 7% E 8%

9. A square $PQRS$ has sides of length x. T is the midpoint of QR and U is the foot of the perpendicular from T to QS. What is the length of TU?



   A $\dfrac{x}{2}$ B $\dfrac{x}{3}$ C $\dfrac{x}{\sqrt{2}}$ D $\dfrac{x}{2\sqrt{2}}$ E $\dfrac{x}{4}$

10. Consider all three-digit numbers formed by using *different* digits from 0, 1, 2, 3 and 5. How many of these numbers are divisible by 6?

    A 4 B 7 C 10 D 15 E 20

11. For what value of x is $\sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} = 2^x$ true?

    A $\frac{1}{2}$ B $1\frac{1}{2}$ C $2\frac{1}{2}$ D $3\frac{1}{2}$ E $4\frac{1}{2}$

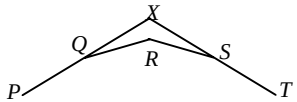
12. Which of the following has the greatest value?

    A $\cos 50°$ B $\sin 50°$ C $\tan 50°$ D $\dfrac{1}{\sin 50°}$ E $\dfrac{1}{\cos 50°}$

13. Suppose that $x - \dfrac{1}{x} = y - \dfrac{1}{y}$ and $x \neq y$. What is the value of xy?

    A 4 B 1 C -1 D -4 E more information is needed

14. P, Q, R, S, T are vertices of a regular polygon. The sides PQ and TS are produced to meet at X, as shown in the diagram, and $\angle QXS = 140°$. How many sides does the polygon have?



    A 9 B 18 C 24 D 27 E 40

15. For how many integers n is $\dfrac{n}{100 - n}$ also an integer?

    A 1 B 6 C 10 D 18 E 100

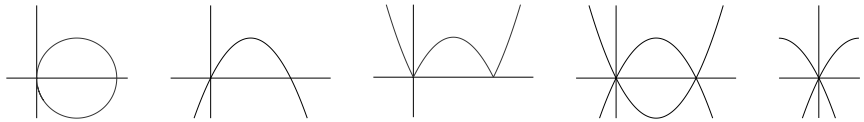
16. The positive numbers x and y satisfy the equations $x^4 - y^4 = 2009$ and $x^2 + y^2 = 49$. What is the value of y?

    A 1 B 2 C 3 D 4 E more information is needed

17. A solid cube is divided into two pieces by a single rectangular cut. As a result, the total surface area increases by a fraction f of the surface area of the original cube. What is the greatest possible value of f?

    A $\dfrac{1}{3}$ B $\dfrac{\sqrt{3}}{4}$ C $\dfrac{\sqrt{2}}{3}$ D $\dfrac{1}{2}$ E $\dfrac{1}{\sqrt{3}}$

18. Which of the following could be part of the graph of the curve $y^2 = x(2 - x)$?



    A B C D E